Year 10 Term 1 Homework

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

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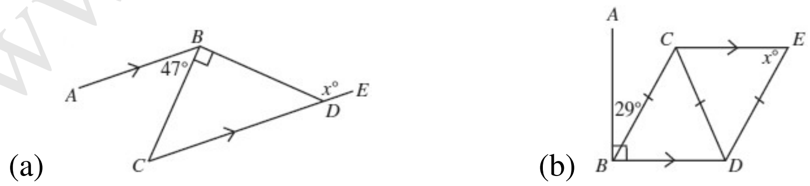
10 Year 10 Term 1 Week 10 Homework Solutions

10.1 Deductive geometry

10.1.1 Basic properties of geometry

- Adjacent angles:
  - have a common vertex
  - have a common ray
  - lie on opposite sides of this common ray.
- Complementary angles have a sum of 90°.
- Supplementary angles have a sum of 180°.
- Angles at a point have a sum of 360°.
- Vertically opposite angles are equal.
- Parallel lines:
- Angle sum of a triangle is 180°.
- The exterior angle of a triangle is equal to the sum of the two interior opposite angles.
- Some other properties of triangles:
  - In a equilateral triangle all angles are 60°
  - In an isosceles triangle, the equal angles are opposite the equal sides.
  - In any triangle, the longest side is opposite the largest angle and the shortest side is opposite the smallest angle.
- The angle sum of a quadrilateral is 360°

Exercise 10.1.1 Find the value of x in each of these, giving reasons.



Solution: *(a)* $x = 47^\circ + 90^\circ = 137^\circ$, *(b)* $x = 90^\circ - 29^\circ = 61^\circ$.

10.1.2 Polygons

- The sum S of the interior angles of any n-sided polygon is given by $S = (n-2) \times 180°$
- The sum S of the exterior angles of any convex polygon is $360°$
- In any regular n-sided convex polygon:
  - each interior angle measures $\frac{180°(n-2)}{n}$
  - each exterior angle measures $\frac{360°}{n}$

Exercise 10.1.2 How many sides have each polygon?

1. *decagon* *10 sides*
2. *nonagon* *9 sides*
3. *dodecagon* *12 sides*
4. *heptagon* *7 sides*
5. *undecagon* *11 sides*

Exercise 10.1.3 Find the sizes of the interior and exterior angles of the following regular polygons:

1. *hexagon:* __________

Solution:

$$\begin{aligned} Interior &= \frac{180(6-2)}{6} = 120° \\ Exterior &= \frac{360°}{6} = 60°. \end{aligned}$$

2. *pentagon:* __________

Solution:

$$\begin{aligned} Interior &= \frac{180(5-2)}{5} = 108° \\ Exterior &= \frac{360°}{5} = 72°. \end{aligned}$$

3. *dodecagon:* __________

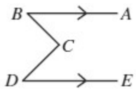
Solution:

$$\begin{aligned} Interior &= \frac{180(12-2)}{12} = 150° \\ Exterior &= \frac{360°}{12} = 30°. \end{aligned}$$

10.1.3 Deductive proofs involving angles

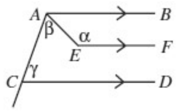
Exercise 10.1.4

1. *Given that BA || DE. Prove that* $\angle BCD = \angle ABC + \angle CDE$.



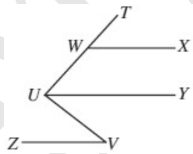
Solution: *Draw a line* $FG||AB$
$\angle ABC = \angle BCF$ and $\angle CDE = \angle FCD$
$\angle BCD = \angle BCF + \angle FCD = \angle ABC + \angle CDE.$

2. *Given that AB || CD || EF. Prove that* $\alpha = \beta + \gamma$.



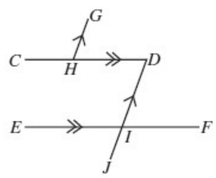
Solution: $\because \angle BAE + \angle\alpha = 180°$, *(co-interior angle)*
$\angle BAE + \angle\beta + \angle\gamma = 180°$ *(co-interior angle)*
$\therefore \angle\alpha = \angle\beta + \angle\gamma.$

3. *WX || UY || ZV and UY bisects* $\angle TUV$. *Prove that* $\angle TWX = \angle UVZ$



Solution: $\angle TUY = \angle YUV$ *UY bisects* $\angle TUY$
$\angle TWX = \angle TUY$ *and* $\angle YUV = \angle UVZ$ *(alternative angle)*
$\therefore \angle YWX = \angle UVZ.$

4. *CD || EF and GH || DJ. Prove that* $\angle CHG = \angle JIF$.



Solution: $\angle JIF = \angle EID$, *opposite angle*
$\angle CDI + \angle DIE = 180°$ *and* $\angle CHG + \angle GHD = 180°$
and $\angle GHD = \angle IDH$ *(Alternative angle)*
$\therefore \angle CHG = \angle JIF.$

10.2 Miscellaneous exercises

Exercise 10.2.1 Find the interior angle sum of a regular polygon that has:

1. *exterior angles measuring* $72°$.

Solution: $\dfrac{360°}{n} = 72, \;\Rightarrow\; n = 5 \;\Rightarrow\; interior = \dfrac{180°(5-2)}{5} = 108°$

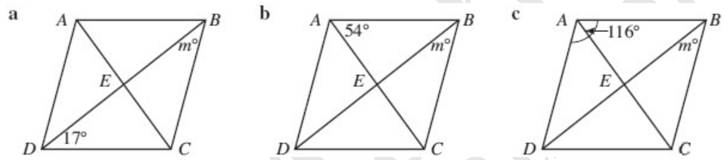
$\therefore\; sum = 180 \times 5 = 540°.$

2. *interior angles measuring* $156°$.

Solution: $interior \;=\; \dfrac{180°(n-2)}{n} = 156°$

$\therefore sum = 156 \times 15 = 2340°.$

Exercise 10.2.2 In each of the following, ABCD is a rhombus. Find the value of m, giving reasons.



Solution: a. $\angle CBD = 17°$ *(base* $\angle s$ *of an isosceles* $\triangle$, $BC = CD$

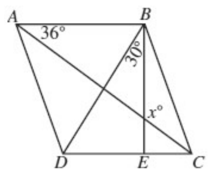
b. $\angle AEB = 90°$ *(diagonals of a rhombus are* $\perp$*)*

$\angle ABE = 36°$ ($\angle$*sum of a* $\triangle$)

$\angle CBE = 36°$ *(diagonals of a rhombus bisects* $\angle s$*)*

c. $\angle ABC = 64°, \;\Rightarrow\; \angle EBC = 32°.$

Exercise 10.2.3 ABCD is a rhombus. $\angle DBE = 30°$**.** $\angle BAC = 36°$**. Find the value of x.**



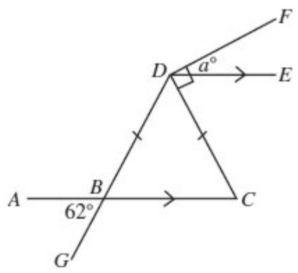
Solution: $\angle D = 180° - 36° \times 2 = 108°$

$\angle DBC = \frac{108}{2} = 54°$ *and* $\angle EBC = 54° - 30° = 24°$

$\therefore \angle x = 180° - 24° - 36\circ = 120°$

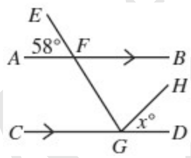
Exercise 10.2.4 Find the value of the pronumeral in each of the following, giving reasons:

1. *In the diagram.* $BD = CD$, $DE||AC$ *and* $CD \perp DF$. *Find the value of a.*



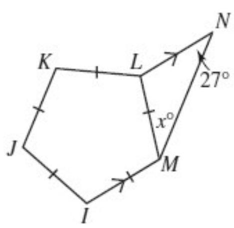
Solution: $\angle DBC = 62°$ *(vertically opposite* $\angle s$*)*
$\angle DCB = 62°$ *(base* $\angle s$ *of an isosceles* $\triangle$, $BD = CD$*)*
$\angle EDC = 62°$ *(alternative* $\angle s$ $DE||AC$*)*
$\angle FDE = 90° - 62° = 28°, \Rightarrow \therefore\ a = 28°$. *(adjacent* $\angle s$ *in a right angle)*

2. *In the diagram,* $AB||CD$ *and* HG *bisects* $\angle FGD$. *Find the value of x.*



Solution: $\angle BFG = 58°$ *(vertically opposite* $\angle s$*)*
$\angle EGD = 122°$ *(Co-interior* $\angle s$ *as* $AB||CD$*)*
$\therefore\ \angle HGD = 122° \div 2 = 61°, \Rightarrow x = 61°$. *(HG bisects* $\angle FGD$*)*

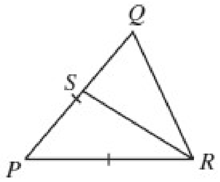
3. *In the diagram IJKLM is a regular pentagon. find the value of x.*



Solution: $\angle IML = 108°$, *(angle in a regular pentagon)*
$\angle MLN = 108°$ *(alternate* $\angle s$*, as* $LN||LM$*),*
$\angle LMN = 45°, \Rightarrow \therefore\ x = 45°$.

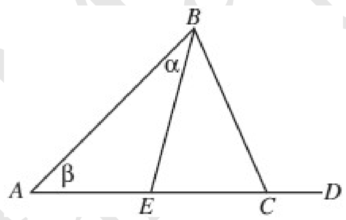
Exercise 10.2.5

1. *In $\triangle PQR$, $PQ = PR$. S is point on PQ such that SR bisects $\angle PRQ$.*
   Prove that $\angle PSR = 3\angle PRS$.



Solution: Let $\angle PRS = \alpha, \Rightarrow \angle QRS = \alpha$, (SR bisects $\angle PRQ$)
$\angle PRQ = 2\alpha$ (sum o f a adjacent angles)
$\angle PQR = \angle PRQ = 2\alpha$, (base angles of isosceles angle)
$\angle PSR = \angle QRS + \angle PQR = 3\alpha$, (exterior angle of $\triangle QRS$)
$\therefore \angle PSR = 3\angle PRS.$

2. *In $\triangle ABC$, AC is produced to D. E is a point on AC such that EB bisects $\angle ABC$.*
   Let $\angle ABE = \alpha$ and $\angle BAC = \beta$.



   (a) *Find the expressions for $\angle BEC$ and $\angle BCD$, giving reasons.*

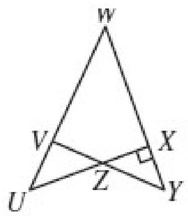
Solution: Let $\angle ABE = \alpha, \ \angle BAC = \beta$
$\angle BEC = \alpha + \beta$ (exterior $\angle$ of $\triangle ABC$)
$\angle EBC = \alpha \angle ABE$ (BE bisects $\angle ABC$)
$\angle BCD = (\alpha + \beta) + \beta = 2\alpha + \beta.$

   (b) *Hence, prove that $\angle BAC + \angle BCD = 2\angle BEC$.*

Solution: $\angle BAC + \angle BCD = \beta + 2\alpha + \beta = 2(\alpha + \beta) = 2\angle BEC.$

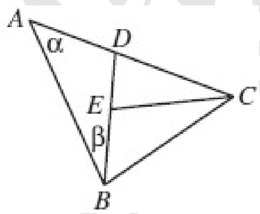
Exercise 10.2.6

1. *In the diagram, $VW = VY$ and $UX \perp WY$. Prove that $\triangle UVZ$ is isosceles.*



Solution: Let $\angle UWY = \alpha$, $\Rightarrow$ $\angle WUX = 90^\circ - \alpha$, ($\angle$Sum of $\triangle UWX$)
$\angle VYW = \alpha$, $\Rightarrow$ $\angle XZY = 90^\circ - \alpha$, ($\angle$Sum of $\triangle XYZ$)
$\angle UZV = 90^\circ - \alpha$, (Vertically opposite $\angle S$)
$\therefore \angle WUX = \angle UZV$, (both equal to $90^\circ - \alpha$)
$\therefore \triangle UV$ is an isosceles.

2. *In $\triangle ABC$, D is a point on AC such that BD bisects $\angle ABC$. E is a point on BD such that $\angle BCE = \angle BAD$. Let $\angle BAC = \alpha$ and $\angle ABD = \beta$.*



(a) *Explain why $\angle BDC = \alpha + \beta$.*

Solution: $\angle BEC = \alpha + \beta$. (exterior $\angle$ of $\triangle ABD$)

(b) *Hence, prove that $CD = CE$.*

Solution: $\angle BCE = \angle BAD = \alpha$, (given)
$\angle DBC = \beta$, (BD bisects $\angle ABC$)
$\angle DEC = \alpha + \beta$ (exterior $\angle$ of $\triangle BCE$)
$\therefore \angle BDC = \angle DEC$, (both equal to $\alpha + \beta$)
$\therefore CD = CE$. (equal sides lie opposite equal $\angle S$)

Exercise 10.2.7 Solve each equation for x:

1. $x(x-1)(4x+1)^2(x^2+1) = 0$

Solution:
$$\begin{cases} x = 0, \\ (x-1) = 0, \Rightarrow \ x = 1, \\ 4x+1 = 0, \Rightarrow \ x = -\frac{1}{4} \\ x^2+1 = 0, \Rightarrow \ x^2 = -1, \Rightarrow \ x = \sqrt{-1} \text{ (invalid)}. \end{cases}$$

2. $x^4 - 64 = 0$

Solution:
$$(x^2-8)(x^2+8) = 0, \Rightarrow \ (x-\sqrt{8})(x+\sqrt{8})(x^2+8) = 0$$
$$\therefore \ x = \pm 2\sqrt{2}, \ \textit{but } x^2+8 \neq 0.$$

3. $x^7 - 3x^5 = 0$

Solution:
$$x^7 - 3x^5 = 0, \Rightarrow \ x^5(x^2-3) = 0,$$
$$\Rightarrow \ x^5(x-\sqrt{3})(x+\sqrt{3}) = 0$$
$$\therefore \ x = 0, \ x = \sqrt{3} \textit{ and } x = -\sqrt{3}.$$

4. $x^4 - 4x^2 = 5$

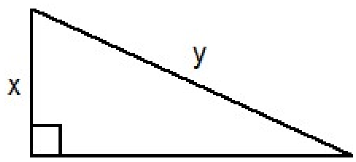
Solution:
$$x^4 - 4x^2 - 5 = 0, \Rightarrow \ (x^2+1)(x^2-5) = 0$$
$$\Rightarrow \ (x^2+1)(x-\sqrt{5})(x+\sqrt{5}) = 0$$
$$\therefore \ x = \pm\sqrt{5} \textit{ but } (x^2+1 \neq 0).$$

5. $4 - 2(x-b) = a + 3$

Solution:
$$4 - 2x + 2b = a + 3 \ \Rightarrow \ -2x = a - 1 - 2b$$
$$\Rightarrow \ x = -\frac{1}{2}(a - 2b - 1)$$

Exercise 10.2.8

1. *Find the area of the triangle shown below in terms of x and y.*



Solution:

$$\begin{aligned} b = \sqrt{y^2 - x^2}, \ \Rightarrow \ A &= \frac{1}{2}xb \\ &= \frac{1}{2}x(\sqrt{y^2 - x^2}) \\ &= \frac{x\sqrt{y^2 - x^2}}{2}. \end{aligned}$$

2. *Reduce to lowest terms:* $\frac{2x^2-3x-2}{10+x-3x^2}$

Solution:

$$\begin{aligned} \frac{2x^2 - 3x - 2}{10 + x - 3x^2} &= \frac{(2x+1)(x-2)}{(5-3x)(2-x)} \\ &= \frac{-2x-1}{5+3x} \end{aligned}$$

3. *Rationalise the denominator:* $\frac{1}{\sqrt{5}+2}$

Solution:

$$\frac{1}{\sqrt{5}+2} = \frac{\sqrt{5}-2}{(\sqrt{5})^2 - 2^2} = \sqrt{5} - 2$$

4. *Simplify:* $\frac{\frac{1}{x+1}+\frac{1}{x}}{\frac{1}{x+1}-\frac{1}{x}}$

Solution:

$$\begin{aligned} \frac{\frac{1}{x+1}+\frac{1}{x}}{\frac{1}{x+1}-\frac{1}{x}} &= \frac{\frac{x}{x(x+1)}+\frac{x+1}{x(x+1)}}{\frac{x}{x(x+1)}-\frac{x+1}{x(x+1)}} \\ &= \frac{\frac{x+x+1}{x(x+1)}}{\frac{x-(x+1)}{x(x+1)}} \\ &= \frac{2x+1}{-1} \\ &= -2x - 1 \end{aligned}$$