Year 10 Term 1 Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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1 Year 10 Term 1 Week 1 Homework Solutions

1.1 Consumer Arithmetic

1.1.1 Simple interest

- **Interest** is money that a bank or other financial institution pays to its customers for lending the bank their money.
- The term **principal** and **capital** refer to an amount of money that has been invested
- Simple interest is the interest that is calculated on the principal only.
- $I = PRT$ where
  - **$P** is the amount invested or borrowed
  - **R** is the percentage rate of interest, i.e. $R = \frac{r}{100}$
  - **T** is the time period of the investment or loan, in years.

Example 1.1.1 Calculate the simple interest earned on an investment of $1280 at 8.6% p.a. over 5 years. Hence find the total balance of the account after 5 years.

Solution: $P = \$1280$, $R = 8.6\% = \frac{8.6}{100} = 0.086$ *and T = 5*

$$I = PRT = \$1280 \times 0.086 \times 5$$
$$= \$550.40$$

The balance = principal + interest

$$= \$1280 + \$550.40 = \$1830.40$$

Example 1.1.2 Calculate the amount of simple interest that must be paid on a loan of $300,000 at 6.8% p.a. over 8 months.

Solution: $P = \$300,000$, $R = 6.8\% = \frac{6.8}{100} = 0.068$ *and* $T = \frac{8}{12}$

$$I = PRT = \$300,000 \times 0.068 \times \frac{8}{12} = \$13,600$$

Exercise 1.1.1 Find the total balance of an account if the following amounts were invested.

1. *$2500 for 5 years at 5.8% p.a* $2500 + 2500 \times 5 \times 5.8\% = \$3225.$
2. *$980 for 8 years at* $5\frac{1}{2}\%$ *p.a* $980 + 980 \times 8 \times 5.5\% = \$1411.20.$
3. *$3800 for 9 months at* $8\frac{1}{4}\%$ *p.a* $3800 + 3800 \times \frac{9}{12} \times 8.25\% = \$4035.13.$
4. *$1820 for 4 months at 4.5% p.a.* $1820 + 1820 \times \frac{4}{12} \times 4.5\% = \$1847.30.$

Exercise 1.1.2 Consolidation

1. *Calculate the simple interest earned on these investments:*

   (a) *\$12,500 at 4.6% p.a for 98 days.* $12500 \times \frac{98}{365} \times 0.046 = \$154.38.$

   (b) *\$8,200 at 1.25% per quarter for 21 months.* $8200 \times \frac{21}{3} \times 1.25\% = \$717.50.$

   (c) *\$13,850 at $4\frac{3}{4}$% p.a. for 202 days.* $13850 \times \frac{202}{365} \times 4.75\% = \$364.08.$

   (d) *\$8040 at 1.35% per quarter for 18 months.* $8040 \times \frac{18}{3} \times 1.35\% = \$651.24.$

   (e) *\$1725 at 7% p.a for 4 months.* $1725 \times \frac{4}{12} \times 7\% = \$40.25.$

2. *Linda borrowed \$8000 from a finance company at 10.5% p.a. simple interest for 3 years to buy some new furniture for her house.*

   (a) *How much interest will Linda pay on her loan?*

   Solution: $\textit{Interest} = 8000 \times 0.105 \times 3 = \$2520.$

   (b) *How much will she repay altogether?*

   Solution: $\textit{Total} = 8000 + 2520 = \$10,520.$

3. *Mary invested a sum of money in a saving account on which simple interest was paid at the rate of 7.5% p.a. After 4 years, Mary had earned \$3600 in interest. How much money did she invest?*

   Solution: $M \times 0.075 \times 4 = \$3600, \Rightarrow M = 3600 \div 0.075 \div 4 = \$12,000.$

4. *Kevin invested \$8500 for 5 years and earned simple interest of \$2762.50. What was the interest rate per annum?*

   Solution: $8500 \times R \times 5 = 2762.50, \Rightarrow R = 2762.50 \div 8500 \div 5 = 0.065,$
   $\therefore R = 6.5\% \ p.a.$

Exercise 1.1.3 Further applications

1. *George invested an amount of money for 6 years at 4.5% p.a. simple interest. What would the interest rate need to be in order for George to earn the same amount of interest in:*

   (a) *2 years*

   Solution:
   $$6 \times 4.5\% = 3 \times R \Rightarrow R = 13.5\% \ p.a.$$

   (b) *3 years*

   Solution:
   $$6 \times 4.5\% = 2 \times R \Rightarrow R = 9\% \ p.a.$$

   (c) *12 years*

   Solution:
   $$6 \times 4.5\% = 12 \times R \Rightarrow R = 2.25\% \ p.a.$$

2. *For how many months would I need to invest \$5000 at 7.2% p.a. simple interest to earn interest of \$150?*

   Solution:
   $$5000 \times 7.2\% \times \frac{N}{12} = \$150,$$
   $$N = 1500 \times 12 \div 5000 \div 7.2\% = 5 \ \textit{months}.$$

3. *Raymond invested \$5250 at 6.4% p.a. simple interest. For how many days was the money invested if he earned \$56.15 interest?*

   Solution:
   $$5250 \times 6.4\% \times \frac{D}{365} = \$56.15$$
   $$D = 56.15 \times 365 \div 5250 \div 6.4\% = 61 \ \textit{days}.$$

4. *John deposited a sum of money into a new credit union account. The account paid simple interest at the rate of 6.5% p.a. and after 3 years he had earned \$165.75 interest. How much money did John invest in the account?*

   Solution:
   $$M \times 6.5\% \times 3 = \$165.75$$
   $$M = 165.75 \div 3 \div 6.5\% = \$850.$$

1.1.2 Compound interest

When compound interest is calculated on an investment, the interest is calculated on the principal as well as on any interest that has been earned previously. The compound interest formula:

$$A = P(1+R)^n \qquad \text{or} \qquad A = P(1+\tfrac{r}{100})^n \text{ where}$$

- **P** is the principal, or the amount invested
- **R** is the interest rate per time period , expressed as a decimal
- **n** is the number of time periods
- **A** is the value of the investment after n time period.

Example 1.1.3 Daniel invested $15,000 at 6.5% p.a. compound interest, with interest compounding annually. Find the value of this investment after 5 years and the amount of interest earned.

Solution: $P = \$15,000,\ R = 0.065$ *and* $n = 5$.
$A = P(1+R)^n = \$15,000(1+0.065)^5 = \$20,551.30.$

Exercise 1.1.4 Use the compound interest formula to find the final value of each investment if the interest is compounded annually (correct to the nearest dollar):

1. *$2000 at 5% p.a. for 4 years* $2000(1+5\%)^4 = \$2431.00.$
2. *$12,000 at 8% p.a. for 5 years* $12000(1+8\%)^5 = \$17632.00.$
3. *$500 at 12% p.a. for 3 years* $500(1+12\%)^3 = \$702.00.$
4. *$4000 at 6% p.a. for 8 years* $4000(1+6\%)^8 = \$6375.00.$

Exercise 1.1.5 Use the compound interest formula to find the final value of each investment if the interest is compounded monthly (correct to the nearest dollar):

1. *$2500 at 12% p.a. for 3 months* $2500(1+1\%)^3 = \$2576.00.$
2. *$8000 at 6% p.a. for 8 months* $8000(1+\frac{6\%}{12})^8 = \$8326.00.$
3. *$25000 at 7.2% p.a. for 18 months* $25000\left(1+\frac{7.2\%}{12}\right)^{18} = \$27842.00.$

Exercise 1.1.6 Consolidation

1. *Ben invests $7000 at 12% p.a. compound interest. Find the value of his investment after 5 years if the interest is compounded (correct to the nearest dollar):*

   (a) *annually*

   Solution: $7000(1+12\%)^5 = \$12336.39 \approx \$12336.$

   (b) *half-yearly*

   Solution: $7000(1+6\%)^{10} = \$12536.$

   (c) *monthly*

   Solution: $7000(1+1\%)^{60} = \$12717.$

2. *Calculate the amount of money, correct to the nearest dollar, that will grow to:*

   (a) *$6766.30 if invested for 7 years at 6% p.a., compounded annually.*

   Solution: $6766.30 \div (1+6\%)^7 = \$44499.9 \approx \$4500.$

   (b) *$2478.60 if invested for 5 years at 12% p.a., compounded half-yearly.*

   Solution: $2478.60 \div \left(1+\frac{12\%}{2}\right)^{10} = \$1384.00.$

   (c) *$1902.36 if invested for 3 years at 8% p.a., compounded quarterly.*

   Solution: $1902.36 \div \left(1+\frac{8\%}{4}\right)^{12} = \$1500.00.$

   (d) *$3868.62 if invested for 2 and a half months at 15% p.a., compounded monthly.*

   Solution: $3868.62 \div \left(1+\frac{15\%}{12}\right)^{2.5} = \$3750.00.$

Exercise 1.1.7 Further applications

1. *Sam earns $860 per week as a mechanical engineer. His boss has agreed to increase his salary by 3% each year. Calculate his annual salary in 5 years time, correct to the nearest dollar.*

   Solution: $860 \times (1+3\%)^5 \times 52 = \$4996.98 \times 52 = \$51,843.$

2. *Benjamin invested a sum of money at 5% p.a., compounded annually. After 5 years his investment had grown to $6940.80. Find the amount of money that he invested, correct to the nearest dollar.*

   Solution: $M \times (1+5\%)^5 = \$6940.80, \Rightarrow M = \$5438.00.$

3. *Paul invested $30,000 at 6% p.a. compounded monthly. Find:*

   (a) *the value of the investment after 5 years.*

   Solution: $30000\left(1+\frac{6\%}{12}\right)^{60} = \$40465.50.$

   (b) *the amount of interest earned.*

   Solution: $40465.50 - 30000 = \$10465.50.$

4. *A finance company invested $500,000 at a daily interest rate of 0.02%. Find:*

   (a) *the value of the investment after 4 weeks.*

   Solution: $500,000(1+0.02\%)^{28} = \$502,807.57.$

   (b) *the amount of interest earned.*

   Solution: $502,807.57 - 500,000 = \$2807.57.$

   (c) *the equivalent annual rate of interest.*

   Solution: $500,000(1+R)^{\frac{4}{52}} = 502,807.57, \Rightarrow (1+R)^{\frac{1}{13}} = 1.0056$

   $1+R = 1.0755 \Rightarrow R = 0.0755 = 7.55\%.$

1.1.3 Depreciation

1. If an item loses value over a period of time, then it is said to depreciate in value.
2. The depreciating value of an item each year is related to its value in the previous year.
3. The rate at which it depreciates if often expressed as a percentage.
4. The depreciation formula: $V = P(1-R)^n$ or $V = P(1-\frac{r}{100})^n$ where
   - P is the original value of the item
   - R is the annual rate of depreciation, expressed as a decimal
   - n is the number of years the item depreciates
   - V is the value of the item after n years.

Example 1.1.4 A PC was purchased in 2006 for $1580 and depreciates by 35% p.a. Find the expected value of the PC in 2008, correct to nearest dollar.

Solution: *P = $1580, R = 0.35 and n = 2*

$V = P(1-R)^n = \$1580(1-0.35)^2 = \$667.55 \doteqdot \$668$

Example 1.1.5 A school purchased a new photocopier. The photocopier depreciated by 25% p.a. and had a value of $3006 after 4 years. Find the purchase price of the photocopier.

Solution: *R = 0.25, n = 4 and V = $3006. Find the P = ? Now* $V = P(1-R)^n$

$3006 = P(1-0.25)^4, \Rightarrow \therefore \ P = \frac{3006}{0.75^4} = \$9500.44 \approx \$9500$

Exercise 1.1.8 Use the depreciation formula to find the value of a computer that was purchased for:

1. *$5000 and depreciated at a rate of 15% p.a. for 5 years*

   Solution: $5000 \times (1-15\%)^5 = \$2218.53$

2. *$2700 and depreciated at a rate of 20% p.a. for 4 years*

   Solution: $2700 \times (1-20\%)^4 = \$1105.92.$

3. *$2000 and depreciated at a rate of 25% p.a. for 3 years*

   Solution: $2000 \times (1-25\%)^3 = \$843.75.$

Exercise 1.1.9 Consolidation

1. *A LCD TV purchased for \$2800 depreciates at the rate of %18 p.a.*

   (a) *What will be the value of the LCD TV after 5 years?*

   Solution:
   $$2800 \times (1 - 18\%)^5 = \$1038.07.$$

   (b) *By how much would the LCD TV depreciate during this time period?*

   Solution:
   $$2800 - 1038.07 = \$1761.93.$$

2. *Ken purchased a share portfolio for \$6000 at the beginning of the year. The shares rose in value by 5% per month for the first 6 months, then fell in value by 5% per month for the next 6 months. Find the value of the shares at the end of the year.*

   Solution:
   $$6000 \times (1 + 5\%)^6 = \$8040.57.$$
   $$8040.57 \times (1 - 5\%)^6 = \$5910.56.$$

3. *A DVD recorder was purchased for \$480 and 2 years later its value had depreciated to \$275. Find the annual rate of depreciation.*

   Solution:
   $$480 \times (1 - R)^2 = 275, \Rightarrow 1 - R = 0.7569,$$
   $$\therefore R = 1 - 0.7569 = 0.2430 = 24.3\%.$$

4. *A car purchased for \$40,000 depreciates at a rate of 20% p.a. After how many years will the car have a value of \$13,107?*

   Solution:
   $$40,000 \times (1 - 20\%)^n = 13,107 \Rightarrow (1 - 20\%)^n = 0.327675$$
   $$0.8^n = 0.3277, \Rightarrow n = \frac{\ln 0.3277}{\ln 0.8} = 5 \text{ years}.$$

1.2 Miscellaneous exercises

Exercise 1.2.1

1. *I want to invest $3500 for 10 years. I have a choice of investing at a simple interest rate of 15.5% p.a. or a compound rate of 10.25% p.a. Which is the better option and by how much?*

   Solution: $3500 + 3500 \times 10 \times 15.5\% = \$8925$. *and* $3500 \times (1 + 10.25\%)^{10} = \$9286.54$
   The secomnd option is beter by $\$9286.54 - \$8925 = \$361.54$.

2. *After being depreciated at 7% p.a. for 12 years, a laser printer is valued at $413. What was its value 12 years ago?*

   Solution: $P \times (1 - 7\%)^{12} = \$413 \Rightarrow P = \$986.63$.

3. *The population of Kangaroo Creek is 2000 and is expected to increase each year by 80% of the pervious year's population. What is the expected population in 5 years' time, correct to the nearest 10?*

   Solution: $2000 \times (1 + 80\%)^5 = 37790$.

4. *Adam invests $20,000 at 12% p.a simple interest while Linda invests $20,000 at 9% p.a. compounded monthly. What is the difference in the value of their investments after: (Answer correct to nearest dollar.)*

   (a) *2 years*

   Solution: $20000 + 20000 \times 12\% \times 2 = \$24800$. *and* $20000 \times (1 + 9\%)^{24} = \$23928.27$.
   Adam has $24800 - 23928 = \$872$ *more*.

   (b) *10 years*

   Solution: $20000 + 20000 \times 12\% \times 10 = \$44000$. *and* $20000\left(1 + \frac{9\%}{12}\right)^{120} = \$49027$.
   Linda has $\$49027 - \$44000 = \$5027$ *more*.

Exercise 1.2.2

1. *A man has 100 notes of \$5 and \$10. He has a total of \$865. How many of each does he have?*

Solution: *Assume they are all \$10 notes,* $\Rightarrow$ $100 \times 10 = \$1000$.
each replacement will have a difference of \$5,
$\therefore$ *the number of \$5 is* $(1000 - 865) \div 5 = 27$, *the number of \$10 is* $100 - 27 = 73$.

2. *Find the value of this continuous product:* $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\left(1+\frac{9}{16}\right)\ldots\left(1+\frac{41}{400}\right)$.

Solution:
$$\begin{aligned}&\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\left(1+\frac{9}{16}\right)\ldots\left(1+\frac{41}{400}\right)\\&= 4\times\frac{9}{4}\times\frac{16}{9}\times\frac{25}{16}\ldots\times\frac{441}{400}\\&= 441.\end{aligned}$$

3. *An isosceles triangle has a perimeter of 25 cm. How many different such triangles can be formed if the sides have to be whole numbers?*

Solution: *Prosible triangles:* $(12,\ 1,\ 12)$, $(11,\ 3,\ 11)$, $(10,\ 5,\ 10)$, $(9,\ 7,\ 9)$, $(8,\ 9,\ 8)$, $(7,\ 11,\ 7)$.
There are a total 6 different isosceles triangles.

4. *Given that* $\frac{x}{x+y} = -2$. *Find the value of* $\frac{x}{y}$.

Solution:
$$\begin{aligned}\frac{x}{x+y} = -2 \ &\Rightarrow\ 2x + 2y = -x\\2y = -3x \ &\Rightarrow \therefore\ \frac{x}{y} = -\frac{2}{3}.\end{aligned}$$

5. *Solve the equation* $\frac{1}{2} - \frac{2}{x} = -1$

Solution:
$$\begin{aligned}\frac{1}{2} - \frac{2}{x} = -1,\ &\Rightarrow\ -\frac{2}{x} = -1 - \frac{1}{2}\\3x = 4\ &\Rightarrow \therefore\ x = 1\frac{1}{3}.\end{aligned}$$

Exercise 1.2.3 Simplify the following expressions:

1. $(64x^6)^{\frac{2}{3}}$

Solution:
$$(64x^6)^{\frac{2}{3}} = [4^3(x^2)^3]^{\frac{2}{3}} = 16x^4.$$

2. $\left(\frac{64p^2}{16q^2}\right)^{\frac{3}{2}}$

Solution:
$$\left(\frac{64p^2}{16q^2}\right)^{\frac{3}{2}} = \frac{8p^3}{q^3}.$$

3. *Simplify* $\frac{2xy^{-3}}{4x^{-1}y^5}$ *writing your answer without negative indies.*

Solution:
$$\frac{2xy^{-3}}{4x^{-1}y^5} = \frac{x^2}{2y^8}.$$

4. $x - \frac{x}{4}$

Solution:
$$x - \frac{x}{4} = \frac{4x}{4} - \frac{x}{4} = \frac{3x}{4}.$$

5. $\frac{3x}{3} + \frac{1-2x}{4}$

Solution:
$$\begin{aligned}\frac{3x}{3} + \frac{1-2x}{4} &= \frac{12x}{12} + \frac{3-6x}{12}\\ &= \frac{6x+3}{12}\\ &= \frac{2x+1}{4}.\end{aligned}$$

6. $\frac{x-2}{3} - \frac{3x-1}{2}$

Solution:
$$\begin{aligned}\frac{x-2}{3} - \frac{3x-1}{2} &= \frac{2x-4-9x+3}{6}\\ &= \frac{-7x-1}{6}.\end{aligned}$$

Exercise 1.2.4 Further applications

1. *If* $x = \frac{3}{5}$, *find the value of* $\frac{2}{x^2}$.

Solution:

$$\frac{2}{x^2} = \frac{2}{(\frac{3}{5})^2} = \frac{2}{\frac{9}{25}} = \frac{50}{9} = 5\frac{5}{9}.$$

2. *I am thinking of a fraction. It it is multiplied by* $\frac{5}{2}$ *and then* $\frac{1}{4}$ *is added, it is equal to* $\frac{7}{12}$ *more then twice the fraction. Find the value of the original fraction.*

Solution:

Let the fraction be N, $\Rightarrow$

$$\frac{5}{2} \times N + \frac{1}{4} = 2N + \frac{7}{12}$$
$$\frac{5N}{2} - \frac{4N}{2} = \frac{7}{12} - \frac{3}{12}$$
$$\frac{N}{2} = \frac{1}{3}$$
$$\therefore \ N = \frac{2}{3}$$

3. *In numbering the pages of a book 318 digits are used. For example, in page 105, three digits are used. How many pages are in the book?*

Solution:

One digit pages: 9 *digits used*

Two digit pages: $2 \times 90 = 180$ *digits used*

Three digits pages: $= 99 + (318 - 180 - 9) \div 3 = 99 + 43 = 142$ *pages.*

4. *Some apples are given out to a group of students. If each receives 5 apples, there are 10 left over, but if each were to receive 6 apples there would be 2 apples too few. Find the number of apples.*

Solution:

Let the number of studnets be N,

then the nubmer of apple will be: $5N + 10$ *or* $6N - 2$.

$\therefore \ 5N + 10 = 6N - 2, \ \Rightarrow \ N = 12.$

$\therefore$ *the number of apples is* : $5 \times 12 + 10 = 70.$