Year 10 Term 2 Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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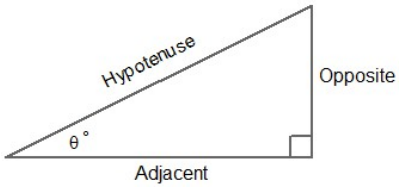
10 Year 10 Term 2 Week 10 Homework

10.1 Trigonometry with Right-angled Triangles

10.1.1 The definition of the trigonometric ration

The definitions of the trigonometric ratio are:



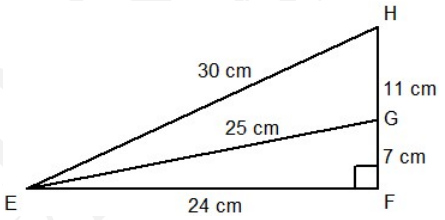
- $\sin\theta = \frac{opposite}{hypotenuse} = \frac{O}{H}$
- $\cos\theta = \frac{adjacent}{hypotenuse} = \frac{A}{H}$
- $\tan\theta = \frac{opposite}{adjacent} = \frac{O}{A}$

An easy way of remembering these important formulae is:

S	O	H	C	A	H	T	O	A
Some	old	houses	can	always	hide	their	old	age

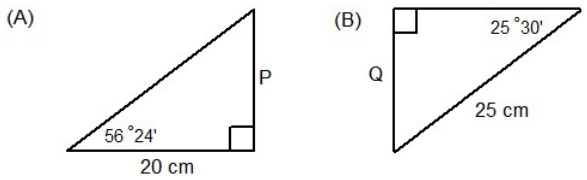
Exercise 10.1.1

1. *Find without simplifying, the value of each ratio below:*



(a) $\sin\angle FEG$ _____ *; (b)* $\cos\angle FEG$ _____ *;*
(c) $\tan\angle FGE$ _____ *; (d)* $\sin\angle FHE$ _____ *;*
(e) $\cos\angle FHE$ _____ *; (f)* $\tan\angle FEH$ _____ *;*

2. *Find the value of each pronumeral, correct to 1 decimal place.*



A $P =$ _____

B $Q =$ _____

10.1.2 Finding the length of a side

Exercise 10.1.2

1. *In $\triangle ABC$, $\angle A = 90°$, $\angle C = 62°45'$ and BC = 70 cm. Find the length of AB, correct to 1 decimal place.*

2. *In $\triangle LMN$, $\angle M = 90°$, $\angle L = 73°21'$ and $LM = 36.7\,cm$. Find the length of LN, correct to 1 decimal place.*

10.1.3 Finding the size of an angle

Exercise 10.1.3

1. *In $\triangle FGH$, $\angle H = 90°$, $GH = 19\,cm$ and $FH = 10\,cm$. Find $\angle F$, correct to the nearest minute.*

2. *In $\triangle IJK$, $\angle I = 90°$, $IK = 12.7\,cm$ and $JK = 15.9\,cm$. Find $\angle K$, correct to the nearest minute.*

10.1.4 Evaluating trigonometric expressions

Exercise 10.1.4

1. *Find the value of $\frac{\tan 76°19'}{\cos 12°36' - \sin 64°10'}$, correct to 2 decimal places.*

2. *If $\tan\theta = 3.6816$, find the acute angle θ, correct to nearest minute.*

10.1.5 Angles of elevation and depression

- The angle of elevation is the angle between the horizontal and the line of sight when the observer is looking upward.
- The angle of depression is the angle between the horizontal and the line of sight when the observer is looking downward.

Exercise 10.1.5

1. *A water pipe runs along the slope of a* $295\,m$ *high hill. The pipe is* $372\,m$ *long. At what angle is the pipe inclined to the horizontal? Answer to the nearest minute.*

2. *A man standing on top of a cliff of height* $165\,m$ *looks down to a boat that is anchored* $115\,m$ *form the base of the cliff. Find the angle of depression of the boat from the top of the cliff, correct to the nearest minute.*

10.1.6 The tangent ratio [$\tan = \frac{\sin\theta}{\cos\theta}$]

Exercise 10.1.6

1. *Find the value of* $\tan\theta$ *in each of the following, where* θ *is an acute angle. Hence find the size of the angle* θ*, correct to the nearest minute.*
   (a) $\sin\theta = \frac{8}{17}$ *and* $\cos\theta = \frac{15}{17}$
   (b) $\sin\theta = 0.7910$ *and* $\cos\theta = 0.6118$
   (c) $\sin\theta = \frac{\sqrt{2}}{3}$ *and* $\cos\theta = \frac{\sqrt{7}}{3}$
2. *In the equations below,* θ *is an acute angle. Express each equation in terms of* $\tan\theta$*, than solve for* θ*, correct to the nearest minute.*
   (a) $\frac{1}{\cos\theta} = \frac{8}{\sin\theta}$
   (b) $\frac{\sqrt{5}}{\cos\theta} = \frac{2}{\sin\theta}$
3. *prove that* $\frac{\sin\theta\cos\theta}{\tan\theta} = \cos^2\theta$.

10.1.7 The complementary results [$\sin\theta = \cos(90° - \theta)$ and $\cos\theta = \sin(90° - \theta)$]

Exercise 10.1.7

1. *Find the value of x in each of these:*
   (a) $\sin 60° = \cos x°$
   (b) $\cos x° = \sin 25°$

2. *Solve each of these equations:*
   (a) $\cos(2x + 56)° = \sin 14°$
   (b) $\sin 40° = \cos(\frac{x}{2})°$
   (c) $\sin(x + 15)° = \cos(x - 6)°$

3. *Simplify the following expressions:*
   (a) $\frac{\cos\theta}{\sin(90° - \theta)}$
   (b) $\sin\theta\cos(90° - \theta)$
   (c) $\sin(90° - \theta) \times \cos(90° - \theta) \times \tan(90° - \theta)$

10.1.8 The exact values

The exact values for the trigonometric ratios are summarised in the table shown below:

θ	$30°$	$45°$	$60°$	θ	$30°$	$45°$	$60°$	θ	$30°$	$45°$	$60°$
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

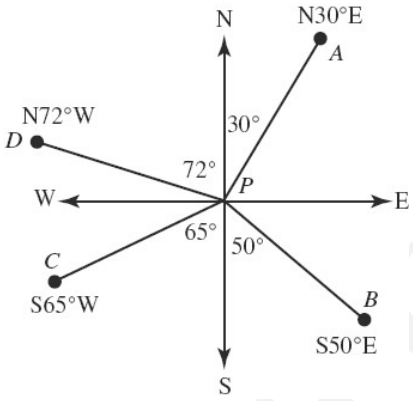
Exercise 10.1.8 Find the exact value of each expression.

1. $\sin 30° + \cos 45° + \tan 60°$
2. $\cos^2 60° - \cos^2 30°$
3. $\cos 45° \cos 30° + \sin 45° \sin 30°$

10.1.9 Compass bearings

- A compass bearing is a deviation involving the four cardinal directions north, south, east and west.
- Compass bearings are always measured from the north or south and towards the east or west.
- A bearing such as NE means $N45°W$. SE means $S45°E$, etc..

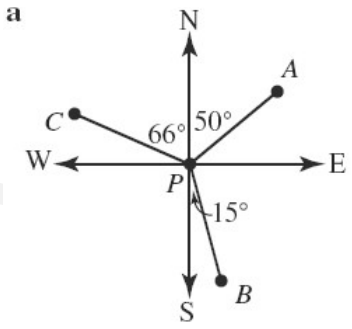
Example 10.1.1



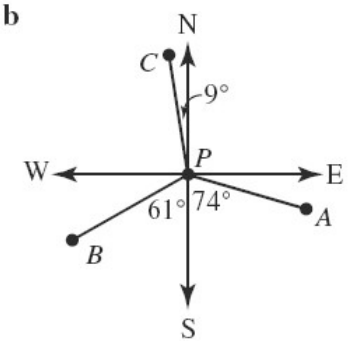
1. *The bearing of A from P is* $N30°E$
2. *The bearing of B from P is* $S50°E$
3. *The bearing of C from P is* $S65°W$
4. *The bearing of D from P is* $N72°W$

Exercise 10.1.9 Find the compass bearings from P of the points A, B and C.

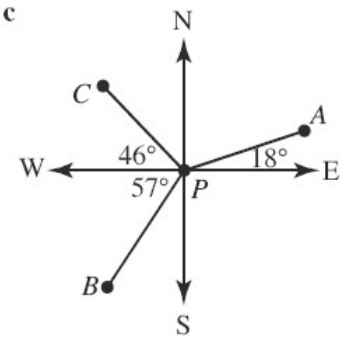
a



b



c



a *Point A* _____ , *Point B* _____ , *Point C* _____

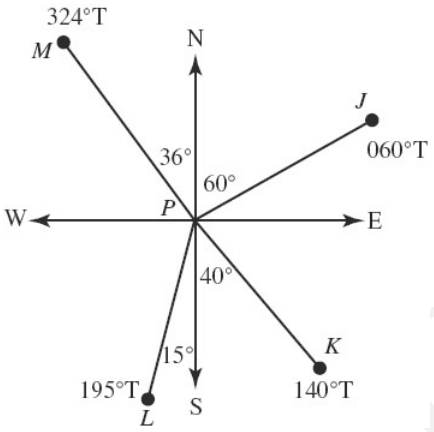
b *Point A* _____ , *Point B* _____ , *Point C* _____

c *Point A* _____ , *Point B* _____ , *Point C* _____

10.1.10 True bearings

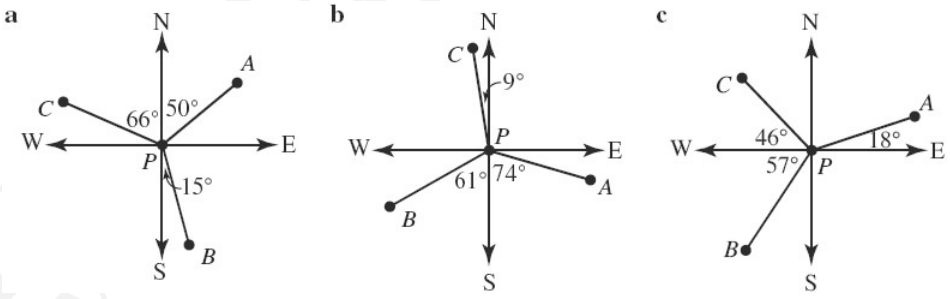
- A true bearing is a deviation from north, measured in a clockwise direction.
- By convention, a true bearing is written using 3 digits.

Example 10.1.2



1. *Point J is* $060°T$
2. *Point K is* $140°T$
3. *Point L is* $195°T$
4. *Point M is* $324°T$

Exercise 10.1.10 Find the true bearings from P of the points X,Y and Z



a *Point A* _____ , *Point B* _____ , *Point C* _____

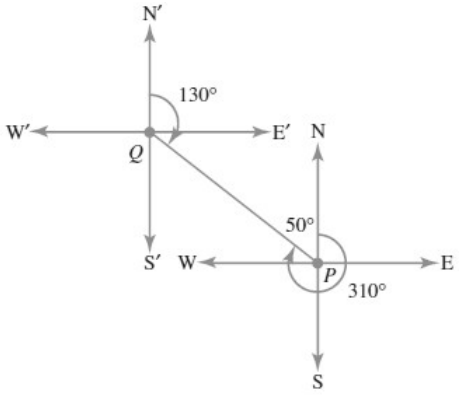
b *Point A* _____ , *Point B* _____ , *Point C* _____

c *Point A* _____ , *Point B* _____ , *Point C* _____

10.1.11 Opposite bearings

- The opposite bearing of B form A is the bearing of A from B.
- To find the bearing of A from B given the bearing of B from A:
  - draw a compass at B and mark on this compass the angle from north around to the ray BA
  - on the compass with centre A, find the acute angle between BA and the north-south axis
  - use parallel line properties to find the required bearing on the compass with centre B.
- Opposite bearings always differ by $180°$

Example 10.1.3 The bearing of Q from P is $310°$. Find the bearing of P from Q.

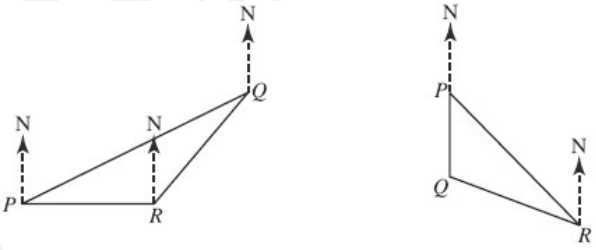


Solution: $\angle NPQ = 360° - 310° = 50°$

$\angle N'QP = 180° - 50° = 130°$ *(co-interior $\angle$s, and $N'Q \| NP$)*

$\therefore$ *The bearing of P from Q is* $130°$.

Exercise 10.1.11 Find the size of $\angle PQR$ for the figures given below:



1. *In the left hand figure, the bearing of Q from P is $034°$ and the bearing of Q from R is $025°$.*

2. *In the right hand figure, the bearing of R from P is $165°$ and the bearing of Q from R is $315°$.*

Exercise 10.1.12 Consolidation

1. *Emma walked from home (H) to a shopping centre (C) on a bearing of* $032°$. *After the shopping, she walked on a bearing of* $122°$ *to a friend's house (F) 850 m due east of her home.*

   (a) Find the value of $\angle HCF$.

   (b) Find the distance between Emma's home and the shopping centre, correct to nearest metre.

2. *David drove from home(H) to the beach (B) on a bearing of* $254°$ *to pick up his children. He then drove to the cinema (C) on a bearing of* $344°$, *which is* $9600\,m$ *due west of his home.*

   (a) Show that $\angle HBC = 90°$.

   (b) Find the distance between the beach and the cinema, correct to nearest metre.

3. *Two cards A and B left home at the same time. Car A travelled due west at* $70\,km/h$ *whilst car B travelled due north at* $90\,km/h$. *Find after* 3 *hours:*

   (a) the distance between two cars, correct to the nearest kilometre.

   (b) the bearing of B from A, correct to nearest degree.

10.1.12 Miscellaneous exercises

Exercise 10.1.13

1. *Given that $V = \frac{1}{3}\pi R^2 H$ and $R > 0$, find R if $V = 2000$ and $H = 12$. Give your answer correct to one decimal place.*

2. *In 2008 Council rates increased by $7\frac{1}{2}\%$. The new rate for a property is \$865. What was the old rate for this property? Give your answer correct to the nearest dollar.*

Exercise 10.1.14 The point P an Q have coordinates (3, -2) and (1, 3) respectively.

1. *The line K has equation $4x + 5y - 2 = 0$. Verify that P lies on K.*

2. *The lines L through Q has gradient $\frac{1}{3}$. Show that the equation of is $x - 3y + 8 = 0$*

3. *The point of intersection of K and L is R. Find the coordinates of R.*

4. *Find the perpendicular distance of P from L. Give your answer in simplest surd form.*