Year 10 Term 2 Homework Worked Solutions

Student Name: ______________	**Grade**: ______________
Date: ______________	**Score**: ______________

Table of contents

This edition was printed on March 14, 2022.

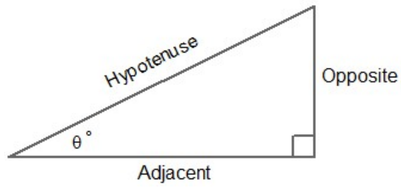
Camera ready copy was prepared with the LaTeX2e typesetting system.

10 Year 10 Term 2 Week 10 Homework Solutions

10.1 Trigonometry with Right-angled Triangles

10.1.1 The definition of the trigonometric ration

The definitions of the trigonometric ratio are:



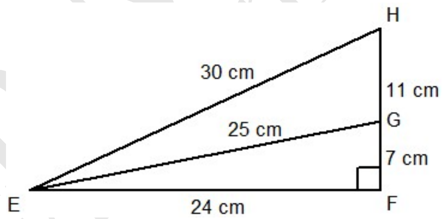
- $\sin\theta = \frac{opposite}{hypotenuse} = \frac{O}{H}$
- $\cos\theta = \frac{adjacent}{hypotenuse} = \frac{A}{H}$
- $\tan\theta = \frac{opposite}{adjacent} = \frac{O}{A}$

An easy way of remembering these important formulae is:

S	O	H	C	A	H	T	O	A
Some	old	houses	can	always	hide	their	old	age

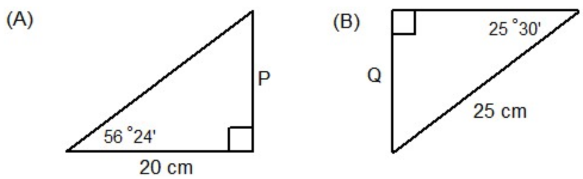
Exercise 10.1.1

1. *Find without simplifying, the value of each ratio below:*



(a) $\sin\angle FEG$ [*Answer:* $\frac{7}{25}$]. ; *(b)* $\cos\angle FEG$ [*Answer:* $\frac{24}{25}$]. ;
(c) $\tan\angle FGE$ [*Answer:* $\frac{24}{7}$]. ; *(d)* $\sin\angle FHE$ [*Answer:* $\frac{24}{30}$]. ;
(e) $\cos\angle FHE$ [*Answer:* $\frac{18}{30}$]. ; *(f)* $\tan\angle FEH$ [*Answer:* $\frac{18}{24}$]. ;

2. *Find the value of each pronumeral, correct to 1 decimal place.*



A $P =$ [*Answer:* $\tan 56°24' = \frac{P}{20} \Rightarrow P = 20 \times \tan 56°24' = 30.1\,cm$].

B $Q =$ [*Answer:* $\sin 25°30' = \frac{Q}{25} \Rightarrow Q = 25 \times \sin 25°30' = 10.8\,cm.$].

10.1.2 Finding the length of a side

Exercise 10.1.2

1. *In $\triangle ABC$, $\angle A = 90°$, $\angle C = 62°45'$ and BC = 70 cm. Find the length of AB, correct to 1 decimal place.*

> ***Solution:*** $$\sin 62°45' = \frac{AB}{70} \Rightarrow AB = 70 \times \sin 62°45' = 62.2\,cm.$$

2. *In $\triangle LMN$, $\angle M = 90°$, $\angle L = 73°21'$ and $LM = 36.7\,cm$. Find the length of LN, correct to 1 decimal place.*

> ***Solution:*** $$\cos 73°21' = \frac{36.7}{LN} \Rightarrow LN = \frac{36.7}{\cos 73°21'} = 128.1\,cm.$$

10.1.3 Finding the size of an angle

Exercise 10.1.3

1. *In $\triangle FGH$, $\angle H = 90°$, $GH = 19\,cm$ and $FH = 10\,cm$. Find $\angle F$, correct to the nearest minute.*

> ***Solution:*** $$\tan F = \frac{19}{10} \Rightarrow \angle F = \tan^{-1}\left(\frac{19}{10}\right) = 62°14'.$$

2. *In $\triangle IJK$, $\angle I = 90°$, $IK = 12.7\,cm$ and $JK = 15.9\,cm$. Find $\angle K$, correct to the nearest minute.*

> ***Solution:*** $$\cos K = \frac{12.7}{15.9} \Rightarrow \angle K = \cos^{-1}\left(\frac{12.7}{15.9}\right) = 36°59'.$$

10.1.4 Evaluating trigonometric expressions

Exercise 10.1.4

1. *Find the value of $\frac{\tan 76°19'}{\cos 12°36' - \sin 64°10'}$, correct to 2 decimal places.*

> ***Solution:*** $$\frac{\tan 76°19'}{\cos 12°36' - \sin 64°10'} = 54.15.$$

2. *If $\tan\theta = 3.6816$, find the acute angle θ, correct to nearest minute.*

> ***Solution:*** $$\angle\theta = \tan^{-1}(3.6816) = 74°48'.$$

10.1.5 Angles of elevation and depression

- The angle of elevation is the angle between the horizontal and the line of sight when the observer is looking upward.
- The angle of depression is the angle between the horizontal and the line of sight when the observer is looking downward.

Exercise 10.1.5

1. *A water pipe runs along the slope of a* $295\,\mathrm{m}$ *high hill. The pipe is* $372\,\mathrm{m}$ *long. At what angle is the pipe inclined to the horizontal? Answer to the nearest minute.*

Solution: $$\sin\theta = \frac{295}{372} \Rightarrow \angle\theta = \sin^{-1}\left(\frac{295}{372}\right) = 52°28'$$

2. *A man standing on top of a cliff of height* $165\,\mathrm{m}$ *looks down to a boat that is anchored* $115\,\mathrm{m}$ *form the base of the cliff. Find the angle of depression of the boat from the top of the cliff, correct to the nearest minute.*

Solution: $$\angle\theta = \tan^{-1}\left(\frac{165}{115}\right) = 55°07'.$$

10.1.6 The tangent ratio [$\tan = \frac{\sin\theta}{\cos\theta}$]

Exercise 10.1.6

1. *Find the value of* $\tan\theta$ *in each of the following, where* θ *is an acute angle. Hence find the size of the angle* θ*, correct to the nearest minute.*

   (a) $\sin\theta = \frac{8}{17}$ *and* $\cos\theta = \frac{15}{17}$ [*Answer:* $\tan\theta = \frac{8}{17} \div \frac{15}{17} = \frac{8}{17} \times \frac{17}{15} = \frac{8}{15} \Rightarrow \angle\theta = 28°04'$].

   (b) $\sin\theta = 0.7910$ *and* $\cos\theta = 0.6118$ [*Answer:* $\tan\theta = \frac{0.791}{0.6118} \Rightarrow \angle\theta = 52°17'.$].

   (c) $\sin\theta = \frac{\sqrt{2}}{3}$ *and* $\cos\theta = \frac{\sqrt{7}}{3}$ [*Answer:* $\tan\theta = \frac{\sqrt{2}}{\sqrt{7}} \Rightarrow \angle\theta = 28°08'.$].

2. *In the equations below,* θ *is an acute angle. Express each equation in terms of* $\tan\theta$*, than solve for* θ*, correct to the nearest minute.*

   (a) $\frac{1}{\cos\theta} = \frac{8}{\sin\theta}$ [*Answer:* $\tan\theta = 8 \Rightarrow \angle\theta = \tan^{-1}(8) = 82°52'.$].

   (b) $\frac{\sqrt{5}}{\cos\theta} = \frac{2}{\sin\theta}$ [*Answer:* $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2}{\sqrt{5}} \Rightarrow \angle\theta = \tan^{-1}(\frac{2}{\sqrt{5}}) = 41°49'$].

3. *prove that* $\frac{\sin\theta\cos\theta}{\tan\theta} = \cos^2\theta$.

Solution: $$LHS = \sin\theta \times \cos\theta \div \frac{\sin\theta}{\cos\theta} = \sin\theta \times \cos\theta \times \frac{\cos\theta}{\sin\theta} = \cos^2\theta = RHS.$$

10.1.7 The complementary results [$\sin\theta = \cos(90^\circ - \theta)$ and $\cos\theta = \sin(90^\circ - \theta)$]

Exercise 10.1.7

1. *Find the value of x in each of these:*
   (a) $\sin 60^\circ = \cos x^\circ$ [*Answer:* $90^\circ - 60^\circ = 30^\circ$].
   (b) $\cos x^\circ = \sin 25^\circ$ [*Answer:* $90^\circ - 25^\circ = 65^\circ$].

2. *Solve each of these equations:*

   (a) $\cos(2x + 56)^\circ = \sin 14^\circ$ [*Answer:* $2x + 56^\circ = 90^\circ - 14^\circ \Rightarrow x = 10^\circ$].

   (b) $\sin 40^\circ = \cos(\frac{x}{2})^\circ$ [*Answer:* $\frac{x}{2} = 50^\circ \Rightarrow x = 100^\circ$].

   (c) $\sin(x + 15)^\circ = \cos(x - 6)^\circ$ [*Answer:* $x + 15^\circ = 90 - (x - 6^\circ) \Rightarrow x = 40^\circ 30'$].

3. *Simplify the following expressions:*

   (a) $\frac{\cos\theta}{\sin(90^\circ - \theta)}$

   Solution:
   $$\frac{\cos\theta}{\cos\theta} = 1$$

   (b) $\sin\theta\cos(90^\circ - \theta)$

   Solution:
   $$\sin\theta \times \sin\theta = \sin^2\theta.$$

   (c) $\sin(90^\circ - \theta) \times \cos(90^\circ - \theta) \times \tan(90^\circ - \theta)$

   Solution:
   $$= \sin(90^\circ - \theta) \times \cos(90^\circ - \theta) \times \frac{\sin(90^\circ - \theta)}{\cos(90 - \theta)}$$
   $$= \sin^2(90^\circ - \theta) = \cos^2\theta.$$

10.1.8 The exact values

The exact values for the trigonometric ratios are summarised in the table shown below:

θ	30°	45°	60°	θ	30°	45°	60°	θ	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Exercise 10.1.8 Find the exact value of each expression.

1. $\sin 30^\circ + \cos 45^\circ + \tan 60^\circ$ [*Answer:* $\frac{1}{2} + \frac{1}{\sqrt{2}} + \sqrt{3} = \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{2\sqrt{3}}{2} = \frac{1+\sqrt{2}+2\sqrt{3}}{2}$].

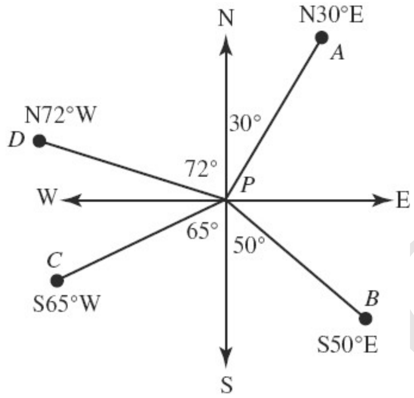
2. $\cos^2 60^\circ - \cos^2 30^\circ$ [*Answer:* $(\frac{1}{2})^2 - (\frac{\sqrt{3}}{2})^2 = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$].

3. $\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$ [*Answer:* $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$].

10.1.9 Compound bearings

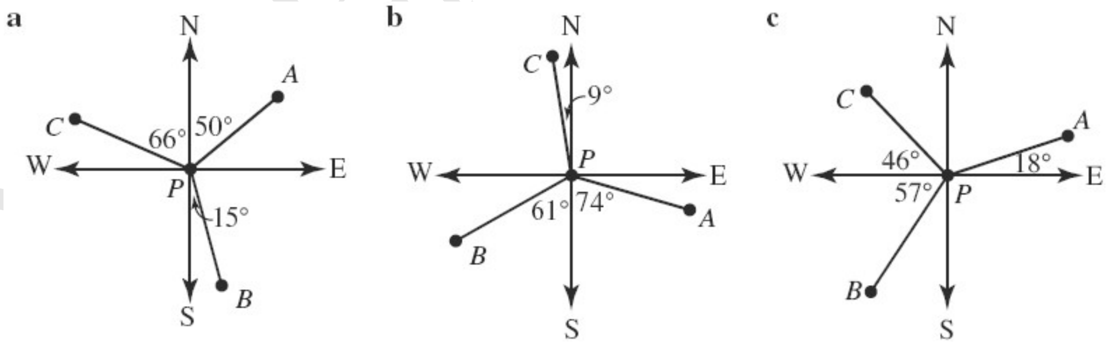
- A compass bearing is a deviation involving the four cardinal directions north, south, east and west.
- Compass bearings are always measured from the north or south and towards the east or west.
- A bearing such as NE means $N45°W$. SE means $S45°E$, etc..

Example 10.1.1



1. *The bearing of A from P is* $N30°E$
2. *The bearing of B from P is* $S50°E$
3. *The bearing of C from P is* $S65°W$
4. *The bearing of D from P is* $N72°W$

Exercise 10.1.9 Find the compass bearings from P of the points A, B and C.



a *Point A* $[= N59°E]$. , *Point B* $[= S15°E]$. , *Point C* $[= N66°W]$.

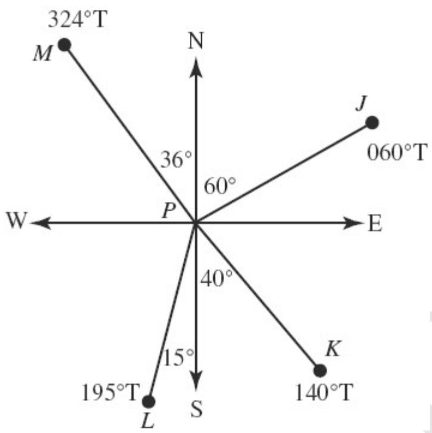
b *Point A* $[= S74°E]$. , *Point B* $[= S61°W]$. , *Point C* $[= N9°W]$.

c *Point A* $[= N72°E]$. , *Point B* $[= S33°W]$. , *Point C* $[= N44°W]$.

10.1.10 True bearings

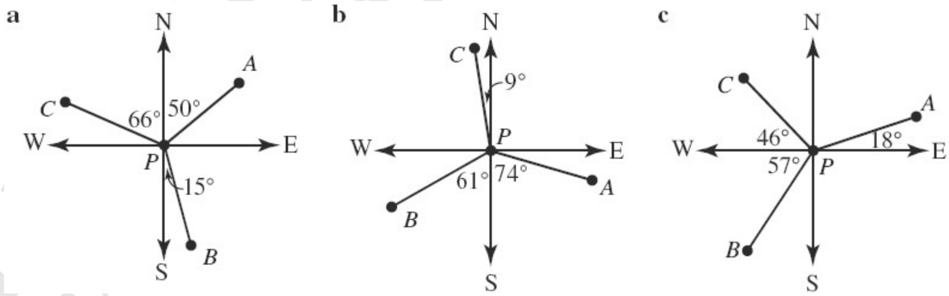
- A true bearing is a deviation from north, measured in a clockwise direction.
- By convention, a true bearing is written using 3 digits.

Example 10.1.2



1. *Point J is* $060°T$
2. *Point K is* $140°T$
3. *Point L is* $195°T$
4. *Point M is* $324°T$

Exercise 10.1.10 Find the true bearings from P of the points X,Y and Z



a *Point A* $[= 050°T]$. , *Point B* $[= 165°T]$. , *Point C* $[= 294°T]$.

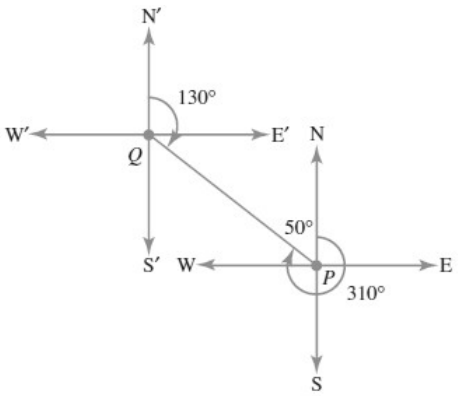
b *Point A* $[= 106°T]$. , *Point B* $[= 241°T]$. , *Point C* $[= 351°T]$.

c *Point A* $[= 072°T]$. , *Point B* $[= 213°T]$. , *Point C* $[= 316°T]$.

10.1.11 Opposite bearings

- The opposite bearing of B form A is the bearing of A from B.
- To find the bearing of A from B given the bearing of B from A:
  - draw a compass at B and mark on this compass the angle from north around to the ray BA
  - on the compass with centre A, find the acute angle between BA and the north-south axis
  - use parallel line properties to find the required bearing on the compass with centre B.
- Opposite bearings always differ by $180°$

Example 10.1.3 The bearing of Q from P is $310°$. Find the bearing of P from Q.

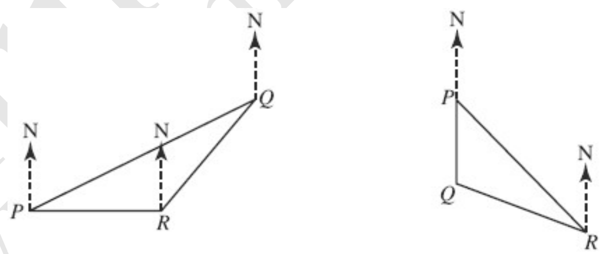


Solution: $\angle NPQ = 360° - 310° = 50°$

$\angle N'QP = 180° - 50° = 130°$ *(co-interior $\angle s$, and $N'Q || NP$)*

$\therefore$ *The bearing of P from Q is* $130°$.

Exercise 10.1.11 Find the size of $\angle PQR$ for the figures given below:



1. *In the left hand figure, the bearing of Q from P is $034°$ and the bearing of Q from R is $025°$.*

   Solution: $\angle PQR = 155° - 146° = 9°$.

2. *In the right hand figure, the bearing of R from P is $165°$ and the bearing of Q from R is $315°$.*

   Solution: $\angle PQR = 315° - 180° = 135°$.

Exercise 10.1.12 Consolidation

1. *Emma walked from home (H) to a shopping centre (C) on a bearing of* 032°. *After the shopping, she walked on a bearing of* 122° *to a friend's house (F) 850 m due east of her home.*

   (a) *Find the value of* $\angle HCF$.

   Solution:
   $$\angle HCF = 32^\circ + (180^\circ - 122^\circ) = 90^\circ.$$
   $$\text{or } \angle HCF = 212^\circ - (180^\circ + 32^\circ) = 90^\circ.$$

   (b) *Find the distance between Emma's home and the shopping centre, correct to nearest metre.*

   Solution: $\angle CHF = 58^\circ, \Rightarrow \cos 58^\circ = \dfrac{x}{850} \Rightarrow \therefore\ x = 850 \times \cos 58^\circ = 450\, m.$

2. *David drove from home(H) to the beach (B) on a bearing of* 254° *to pick up his children. He then drove to the cinema (C) on a bearing of* 344°, *which is* $9600\, m$ *due west of his home.*

   (a) *Show that* $\angle HBC = 90^\circ$.

   Solution:
   $$\angle a = 270^\circ - 254^\circ = 16^\circ,\ \angle b = 74^\circ,\ \angle c = 16^\circ.$$
   $$\angle HBC = \angle b + \angle c = 74^\circ + 16^\circ = 90^\circ.$$

   (b) *Find the distance between the beach and the cinema, correct to nearest metre.*

   Solution: $\sin a = \dfrac{BC}{CH} \Rightarrow BC = CH \times \sin 16^\circ = 9600 \times \sin 16^\circ = 2646\, m$

3. *Two cards A and B left home at the same time. Car A travelled due west at* $70\, km/h$ *whilst car B travelled due north at* $90\, km/h$. *Find after* 3 *hours:*

   (a) *the distance between two cars, correct to the nearest kilometre.*

   Solution:
   $$a = 3 \times 70 = 210\, km,\ \ b = 3 \times 90 = 270\, km.$$
   $$c = \sqrt{a^2 + b^2} = \sqrt{210^2 + 270^2} = 342\, km.$$

   (b) *the bearing of B from A, correct to nearest degree.*

   Solution:
   $$\tan\theta = \frac{b}{a} = \frac{270}{210} \Rightarrow \theta = 52^\circ 07' \approx 52^\circ$$
   $$\therefore\ \text{the bearing of B from A} = 90^\circ - 52^\circ = 38^\circ$$

10.1.12 Miscellaneous exercises

Exercise 10.1.13

1. *Given that* $V = \frac{1}{3}\pi R^2 H$ *and* $R > 0$*, find R if* $V = 2000$ *and* $H = 12$*. Give your answer correct to one decimal place.*

Solution: $V = \frac{1}{3}\pi R^2 H \;\Rightarrow\therefore\; 2000 = \frac{1}{3}\pi R^2(12) \;\Rightarrow\; R = \frac{2000}{4\pi} = 12.6156 \approx 12.6$

2. *In 2008 Council rates increased by* $7\frac{1}{2}\%$*. The new rate for a property is \$865. What was the old rate for this property? Give your answer correct to the nearest dollar.*

Solution: *old rate +* $7\frac{1}{2}\%$ *of old rate = new rate* $\;\Rightarrow\; x + 7.5\%x = 865$

$\Rightarrow\; 1.075x = 865 \;\therefore\; x = \$804.65 = \$805$

Exercise 10.1.14 The point P an Q have coordinates (3, -2) and (1, 3) respectively.

1. *The line K has equation* $4x + 5y - 2 = 0$*. Verify that P lies on K.*

Solution: $LHS = 4x + 5y - 2 = 4 \times (3) + 5 \times (-2) - 2 = 0 = RHS \;\Rightarrow\therefore$ *P lies on K.*

2. *The lines L through Q has gradient* $\frac{1}{3}$*. Show that the equation of is* $x - 3y + 8 = 0$

Solution:

- *Equation of line L through Q(1, 3) with gradient* $\frac{1}{3}$ *is given by:*

$y - y_1 = m(x - x_1) \;\Rightarrow\; y - 3 = \frac{1}{3}(x - 1) \;\Rightarrow\; x - 3y + 8 = 0$

3. *The point of intersection of K and L is R. Find the coordinates of R.*

Solution: $\begin{cases} K: 4x + 5y - 2 = 0 \ldots(1) \\ L: x - 3y + 8 = 0 \quad \ldots(2) \end{cases} \;\Rightarrow\; y = 2,\; x = -2,\; \Rightarrow\therefore\; R \equiv (-2, 2).$

4. *Find the perpendicular distance of P from L. Give your answer in simplest surd form.*

Solution: $D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} = \frac{|(1)(3) + (-3)(-2) + 8|}{\sqrt{1^2 + (-3)^2}} = \frac{17}{\sqrt{10}} = \frac{17\sqrt{10}}{10}$ *units.*