Year 10 Term 2 Homework Worked Solutions

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

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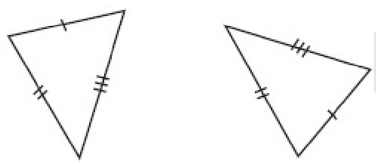
1 Year 10 Term 2 Week 1 Homework Solutions

1.1 Deductive geometry

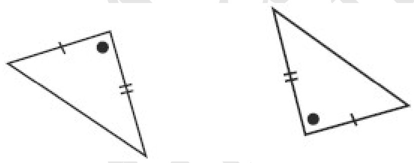
1.1.1 Congruent triangles

If two triangles are congruent, then:

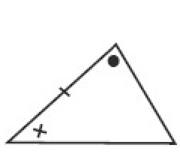
- the matching sides are equal in length.
- the matching angles are equal in size.
- the figures are equal in area.
- If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent (SSS).



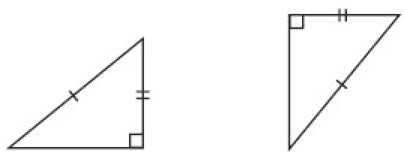
- If two sides and the included angle of one of triangle are equal to two sides and the included angle of another triangle, then the two triangles are congruent (**SAS**).



- If two angles and one side of one triangle are equal to two angles and the matching side of another triangle, then the two triangles are congruent (**AAS**).



- If the hypotenuse and a second side of one right-angled triangle are equal to the hypotenuse and a second of another right-angled triangle, then the two triangles are congruent(**RHS**).

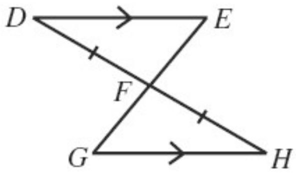


To prove that two triangles are congruent:

- Identify the triangles that are being used in the proof and name the three pairs of equal sides or angles.
- Name the congruent triangles, giving the vertices of the triangles in the matching order, and state the congruence test used.

Exercise 1.1.1

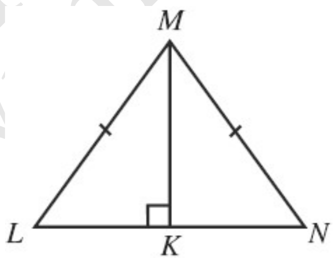
1. *Prove that* $\triangle DEF \equiv \triangle HGF$.



Solution:

$$\angle EDF = \angle GHF, \text{ (alternative angles)}$$
$$\angle EFD = \angle GFH \text{ (vertically opposite)}$$
$$\angle DEF = \angle FGH \text{ (Alternative angles)}$$
$$DF = FH, \Rightarrow \therefore \triangle DEF \equiv \triangle HGF \ (AAS).$$

2. *Prove that* $\triangle MLK \equiv \triangle MNK$.

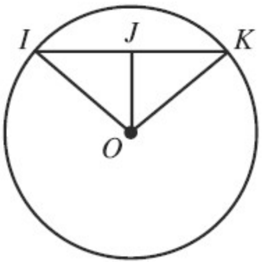


Solution:

$$ML = MN \text{ (Given)}$$
$$MK = MK \text{ (Commont side)}$$
$$\angle MKL = \angle MKN = 90^\circ \text{ (Given)}$$
$$\therefore \ \triangle MLK = \triangle MNK \ (RHS).$$

Exercise 1.1.2

1. *O is the centre of the circle and $OJ \perp IK$. Prove that OJ bisects $\angle IOK$.*



Solution:

$OI = OK$ (equal radio)

OJ is a common side.

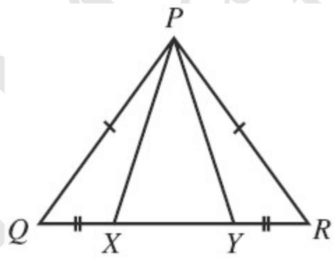
$\angle OJI = \angle OJK = 90^\circ$ $(OJ \perp IK)$

$\because \triangle OIJ \equiv \triangle OKJ$ (RHS)

$\therefore \angle IOJ = \angle KOJ$ (matching angles)

$\therefore OJ$ bisects $\angle IOK$.

2. *In the isosceles triangle PQR, $PQ = PR$. $QX = RY$.*



(a) *Prove that $\triangle PQX \equiv \triangle PRY$.*

Solution:

$PQ = PR$ (given)

$QX = RY$ (given)

$\angle PQX = \angle PRY$ (Base angles of isosceles)

$\therefore \triangle PQX \equiv \triangle PRY$ (SAS).

(b) *Hence, show that $\triangle PXY$ is isosceles.*

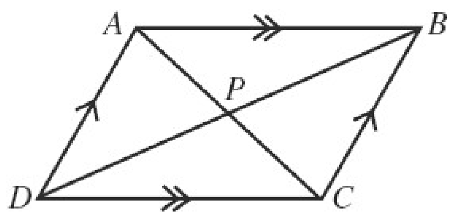
Solution:

$PX = PY$ (matching sides of congruent triangles)

$\therefore \triangle PXY$ is an isosceles triangle.

1.2 Deductive proofs involving quadrilaterals

Exercise 1.2.1 ABCD is a parallelogram. The diagonals AC and BD meet at P.



1. *Prove that* $\triangle APB \equiv \triangle CPD$.

Solution:

$\angle ABP = \angle PDC$ *(alternative angles, AB||DC)*

$\angle BAP = \angle PCD$ *(alternative angles, AB||DC)*

$AB = DC$ *Opposite sides of a parallelogram.*

$\therefore\ \triangle APB \equiv \triangle CPD$ (AAS).

2. *Hence show that* $AP = PC$ *and* $DP = PB$.

Solution:

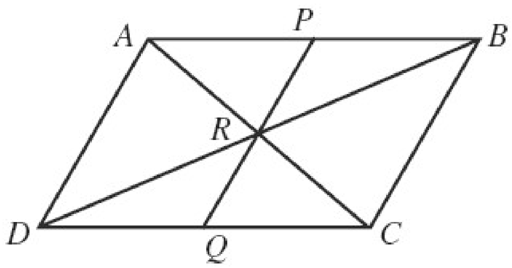
$AP = PC$ *(Matching sides of congruent triangles)*

$BP = PD$ *(Matching sides of congruent triangles.)*

3. *What property of a parallelogram have you proven?*

Solution: *diagonals of a parallelogram bisect each other.*

Exercise 1.2.2 ABCD is a parallelogram. The diagonals AC and BD meet at R. A line PQ is drawn through R, where P lies on AB and Q lies on DC.



1. *Prove that* $\triangle BPR \equiv \triangle DQR$.

Solution:

$\angle PBR = \angle RDC$ (alternative angles AB||DC)

$\angle BPR = \angle PQD$ (alternative angles AB||BC)

$BR = RD$ (AC bisects BD)

$\therefore\ \triangle BPR \equiv \triangle DQR\ (AAS)$.

2. *Hence show that* $PB = DQ$ *and* $AP = QC$.

Solution:

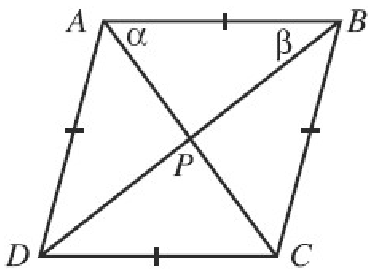
$PB = DQ$ (matching sides of congruent triangles.)

$AB = DC$ (opposite sides of a parallelogram)

$AB - PB = DC - DQ$

$\therefore\ AP = QC$.

Exercise 1.2.3 ABCD is a rhombus. The diagonals AC and BD meet at P. Let $\angle CAB = \alpha$ and $\angle ABD = \beta$.



1. *Explain why $\angle BCA = \alpha$ and $\angle CBD = \beta$.*

> ***Solution:***
> $\angle BCA = \alpha$ *(base angles of isosceles trinagle, AB = BC)*
> $\angle CBD = \beta$ *(diagonal of a rhombus bisect the angle at the vertex.)*

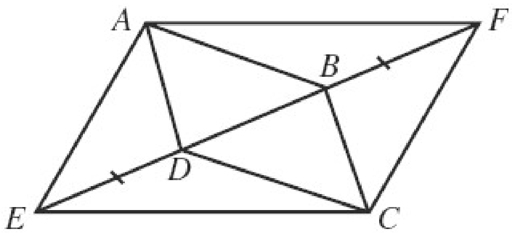
2. *Find the value of $\alpha + \beta$.*

> ***Solution:***
> $$2\alpha + 2\beta = 180^\circ, \Rightarrow \therefore \alpha + \beta = 90^\circ$$

3. *Hence, explain why $AC \perp BD$.*

> ***Solution:***
> $\angle BPC = \alpha + \beta,$ *(exterior angle of $\triangle ABP$)*
> $\therefore \angle BPC = 90^\circ$
> $\therefore AC \perp BD$. *(diagonals ofa rhombus are perpendicular to each other.)*

Exercise 1.2.4 ABCD is a parallelogram. BD **is produced to** E **and** DB **is produced to** F **such that** $DE = BF$**.**



1. *Show that* $\angle FBC = \angle ADE$.

Solution:

Let $\angle DBC = \alpha$, $\Rightarrow$ $\angle ADB = \alpha$ (alternative angles, $AB||DC$)

$\angle FBC = 180° - \alpha$, and $\angle ADE = 180° - \alpha$

$\therefore$ $\angle FBC = \angle ADE$ (both equal to $180° - \alpha$)

2. *Prove that* $\triangle FBC \equiv \triangle EDA$.

Solution:

$\angle FBC = \angle ADE$ (proven above)

$BC = AD$ (opposite sides of a parallelogram)

$BF = ED$ (given)

$\therefore$ $\triangle FBC \equiv \triangle EDA$ (SAS).

3. *Hence prove that AFCE is a parallelogram.*

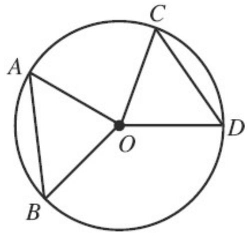
Solution: $\angle BFC = \angle DEA$ (matching angles of congruent triangles)

$\therefore$ $FC||AE$ (Alternative angles are equal)

$\therefore$ $AFCE$ is a parallelogram. (one pair opposite side equal and parallel.)

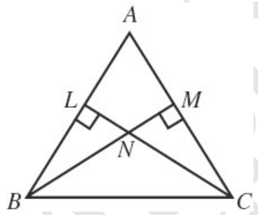
1.3 Miscellaneous exercises

Exercise 1.3.1 O is the centre of the circle an $AB = CD$. Prove that $\angle AOB = \angle COD$.



Solution:	$OA = OD$ *(equal radii)*
	$OB = OC$ *(equal radii)*
	$AB = CD$ *(given)*
	$\therefore\ \triangle AOB \equiv \triangle COD\ (SSS)$.
	$\angle AOB = \angle COD$ *(matching angles of congruent angles).*

Exercise 1.3.2 In the isosceles triangle ABC. AB = AC. $CL \perp AB$ and $BM \perp AC$.



1. *Prove that* $\triangle BLC \equiv \triangle CMB$.

Solution:	$\angle ABC = \angle ACB$ *(base angles of isosceles triangle.)*
	$\angle BLC = \angle CMB = 90^\circ\ (CL \perp AB,\ BM \perp AC.)$
	BC is a common side $\therefore\ \triangle BLC \equiv \triangle CMB\ (AAS)$.

2. *Prove that* $\triangle BLN \equiv \triangle CMN$.

Solution:	$BL = CM$ *(matching sides of congruent triangles)*
	$\angle BLN = \angle CMN = 90^\circ\ (CL \perp AB,\ BM \perp AC.)$
	$\angle BNL = \angle CNM$ *(vertically opposite angles)*
	$\therefore\ \triangle BLN \equiv \triangle CMN\ (AAS)$.

3. *Hence show that* $LN = MN$

Solution:	$LN = MN$ *(Matching sides of congruent triangles).*

Exercise 1.3.3

1. *The length of a rectangle is 8 cm greater than its breadth. If the area of the rectangle is 345*cm^2*, find the perimeter of the rectangle.*

> ***Solution:*** $\begin{cases} x+8=y\ldots(1) \\ xy=345\ldots(2) \end{cases} \Rightarrow x=15\,cm, \text{ and } y=23\,cm$
>
> $\therefore\ P=(15+23)\times 2=76\,cm.$

2. *The product of two positive integers is 112 and the larger number is 6 more than the smaller number. Find the numbers.*

> ***Solution:*** $\begin{cases} A\times B=112\ldots(1) \\ A=B+6\ldots(2) \end{cases} \Rightarrow A(A-6)=112, \Rightarrow A^2-6A-112=0.$
>
> $\therefore\ A=8, \text{ and } B=14.$

3. *Solve the following equations, giving the solutions correct to 2 decimal places where necessary.*

   (a) $x^2-25=2x+10$

> ***Solution:*** $x^2-25=2x+10 \Rightarrow x^2-2x-35=0 \Rightarrow (x-7)(x+5)=0$
>
> $\therefore\ x=7, \text{ or } x=-5.$

   (b) $x+\frac{16}{x}=8$

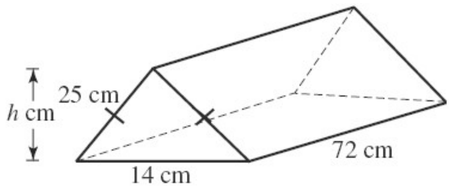
> ***Solution:*** $x+\dfrac{16}{x}=8 \Rightarrow x^2+16=8x \Rightarrow x^2-8x+16=0,$
>
> $\Rightarrow (x-4)^2 \Rightarrow \therefore\ x=4.$

   (c) $\frac{3}{x}-\frac{7x}{2}=4$

> ***Solution:*** $\dfrac{3}{x}-\dfrac{7x}{2}=4 \Rightarrow 6-7x^2=8x \Rightarrow 7x^2+8x-6=0$
>
> $$a=7, b=8, \text{ and } c=-6 \Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
> $$=\frac{-8\pm\sqrt{8^2-4\times 7\times(-6)}}{2\times 7},$$
> $$\therefore\ x=\frac{-8\pm 2\sqrt{58}}{14}=\frac{-4\pm\sqrt{58}}{7} \text{ or } x_1=0.52,\ x_2=-1.66.$$

Exercise 1.3.4

1. *Find the value of the pronumeral in the figure. Hence calculate the surface area.*



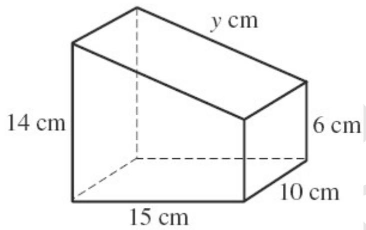
Solution:

$h = \sqrt{25^2 - 7^2} = 24\,cm$

$A_1 = \frac{1}{2} \times 24 \times 14 = 168\,cm^2$

$A = 2 \times 168 + 2 \times 25 \times 72 + 14 \times 72 = 4944\,cm^2.$

2. *Find the value of the pronumeral in the figure. Hence calculate the surface area.*



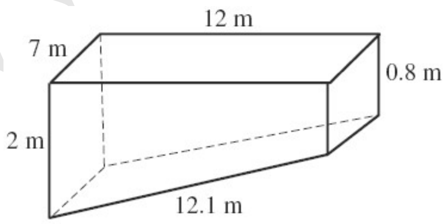
Solution:

$y = \sqrt{15^2 + 8^2} = 17\,cm$

$A_1 = \frac{1}{2}(6 + 14) \times 15 = 150\,cm^2.$

$A = 2 \times 150 + 6 \times 60 + 14 \times 10 + 10 \times 15 + 17 \times 10 = 820\,cm^2.$

3. *Find the surface area and the volume of the figure shown below:*



Solution:

$A_1 = \frac{1}{2}(0.8 + 2) \times 12 = 16.8\,m^2$

$A = 0.8 \times 7 + 16.8 \times 2 + 7 \times 2 + 7 \times 12 + 7 \times 12.1 = 221.9\,m^2.$

$V = 16.8 \times 7 = 117.6\,m^3.$