

## Year 11 Math Homework

<b>Student Name:</b> _____	<b>Grade:</b> _____
<b>Date:</b> _____	<b>Score:</b> _____

### Table of Contents

<b>10 Year 11 Topic 10 — Integration (Part 1 The Indefinite Integral)</b>	<b>1</b>
10.1 Rules of Differentiation and Integration . . . . .	1
10.2 The Indefinite Integral (Primitive Function) . . . . .	2
10.3 Integration Involving Logarithms . . . . .	4
10.4 Integration Involving Trigonometry . . . . .	6
10.5 Practical Exam Questions . . . . .	7
10.6 Miscellaneous exercise . . . . .	9

This edition was printed on February 25, 2022 with **Worked Solutions**.

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## 10 Year 11 Topic 10 — Integration (Part 1 The Indefinite Integral)

### 10.1 Rules of Differentiation and Integration

$f(x)$	$f'(x)$	$f(x)$	$\int f(x).dx$
$kx$	$k$	$k$	$kx + c$
$x^n$	$nx^{n-1}$	$x^n$	$\frac{x^{n+1}}{n+1} + c$
$ax^n + bx + c$	$anx^{n-1} + b$	$ax^n$	$\frac{ax^{n+1}}{(n+1)} + c$
$f(h(x))$	$f'(h(x)).h'(x)$	$(ax + b)^n$	$\frac{(ax+b)^{n+1}}{a(n+1)} + c$
$g(x).h(x)$	$g'(x).h(x) + g(x).h'(x)$	$f'(x).(f(x))^n$	$\frac{1}{n+1}(f(x))^{n+1} + c$
$y = uv$	$y' = uv' + u'v$		
$\frac{g(x)}{h(x)}$	$\frac{g'(x).h(x) - g(x).h'(x)}{[h(x)]^2}$		
$y = \frac{u}{v}$	$y' = \frac{u'v - uv'}{v^2}$		
$e^x$	$e^x$	$e^x$	$= e^x + c$
$e^{kx}$	$ke^{kx}$	$e^{kx}$	$\frac{1}{k}e^{kx} + c$
$\ln x$	$\frac{1}{x}$	$\frac{1}{x}$	$\ln x + c$
$\ln kx$	$\frac{k}{x}$	$\frac{k}{x}$	$\frac{1}{k} \ln x + c$
$\ln f(x)$	$\frac{f'(x)}{f(x)}$	$\frac{f'(x)}{f(x)}$	$\ln f(x) + c$
$\sin x$	$\cos x$	$\sin x$	$-\cos x + c$
$\sin kx$	$k \cos kx$	$\sin kx$	$-\frac{1}{k} \cos kx + c$
$\cos x$	$-\sin x$	$\cos x$	$\sin x + c$
$\cos kx$	$-k \sin kx$	$\cos kx$	$\frac{1}{k} \sin kx + c$
$\tan x$	$\sec^2 x$	$\sec^2 x$	$\tan x + c$
$\tan kx$	$k \sec^2 kx$	$\sec^2 kx$	$\frac{1}{k} \tan kx + c$
$\sin^{-1} \frac{x}{a}$	$\frac{1}{\sqrt{a^2 - x^2}}$	$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1} \frac{x}{a} + c$
$\cos^{-1} \frac{x}{a}$	$-\frac{1}{\sqrt{a^2 - x^2}}$		
$\tan^{-1} \frac{x}{a}$	$\frac{a}{a^2 + x^2}$	$\frac{1}{a^2 + x^2}$	$\frac{1}{a} \tan^{-1} \frac{x}{a} + c$

## 10.2 The Indefinite Integral (Primitive Function)

### Definition:

$$\int (ax^n + x^n + b) dx = \frac{ax^{n+1}}{n+1} + \frac{x^{n+1}}{n+1} + bx + c, \text{ where } a, b, c \text{ are constants, } n \neq -1$$

### Definition:

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c, \text{ where } a, b, c \text{ are constants, } n \neq -1$$

### Example 10.2.1 Find the following indefinite integrals:

1.  $\int (3x^2 + 2x - 4) dx$

$$\text{Solution: } \int (3x^2 + 2x - 4) dx = \frac{3x^3}{3} + \frac{2x^2}{2} - 4x + c = x^3 + x^2 - 4x + c$$

2.  $\int (3x + 2)^3 dx$

$$\text{Solution: } \int (3x + 2)^3 dx = \frac{(3x+2)^4}{3(3+1)} + c = \frac{(3x+2)^4}{12} + c$$

3.  $\int \sqrt{x} dx$

$$\text{Solution: } \int \sqrt{x} dx = \int x^{(\frac{1}{2})} dx = \frac{x^{(\frac{1}{2}+1)}}{\frac{1}{2}+1} + c = \frac{2}{3}x^{\frac{3}{2}} + c$$

4.  $\int (6 - \frac{1}{x^4}) dx$

$$\text{Solution: } \int (6 - \frac{1}{x^4}) dx = \int (6 - x^{-4}) dx = 6x - \frac{x^{(-4+1)}}{(-4+1)} + c = 6x + \frac{x^{-3}}{3} + c = 6x + \frac{1}{3x^3} + c$$

**Exercise 10.2.1** Find the following indefinite integrals:

1.  $\int \frac{4x-1}{x^3} dx$

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2.  $\int \frac{4}{\sqrt[3]{x}} dx$

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3.  $\int \frac{1}{\sqrt{x+1}} dx$

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4.  $\int (3x-2)(x+3) dx$

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5.  $\int (2x-1)^3 dx$

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### 10.3 Integration Involving Logarithms

**Definition:**

$$\int \frac{1}{x} dx = \log_e x + c$$

$$\int \frac{f'(x)}{f(x)} dx = \log_e [f(x)] + c$$

**Definition:**

$$\int e^x dx = e^x + c; \quad \int e^{kx} dx = \frac{1}{k} e^{kx} + c$$

**Example 10.3.1** Find the following indefinite integrals:

1.  $\int \frac{1}{3x+1} dx$

**Solution:** Here  $f(x) = 3x + 1$  so  $f'(x) = 3$ , We will have to adjust the numerator:

$$\int \frac{1}{3x+1} dx = \int \frac{1}{3} \times \frac{3}{3x+1} dx = \frac{1}{3} \int \frac{3}{3x+1} dx = \frac{1}{3} \log_e (3x+1) + c$$

2.  $\int \frac{2x}{x^2-1} dx$

**Solution:** Here  $f(x) = x^2 - 1$  so  $f'(x) = 2x$ , No adjustments are necessary:

$$\int \frac{2x}{x^2-1} dx = \log_e (x^2 - 1) + c$$

3.  $\int \frac{x}{x^2+2} dx$

**Solution:** Here  $f(x) = x^2 + 2$  so  $f'(x) = 2x$ , we will have to adjust the numerator:

$$\int \frac{x}{x^2+2} dx = \frac{1}{2} \int \frac{2x}{x^2+2} dx = \frac{1}{2} \ln (x^2 + 2) + c$$

**Exercise 10.3.1 Find the following indefinite integrals:**

1.  $\int \frac{2x+1}{x^2+x} dx$

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2.  $\int \frac{x}{4-x^2} dx$

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3.  $\int \frac{4x^2}{x^3+1} dx$

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4.  $\int \frac{4x}{x^2-1} dx$

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5.  $\int \left(\frac{1}{x^3} + \frac{1}{x}\right) dx$

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## 10.4 Integration Involving Trigonometry

**Definition:**

$$\int \sin x \, dx = -\cos x + c; \quad \int \sin kx \, dx = -\frac{1}{k} \cos kx + c$$

$$\int \cos x \, dx = \sin x + c; \quad \int \cos kx \, dx = \frac{1}{k} \sin kx + c$$

$$\int \sec^2 x \, dx = \tan x + c; \quad \int \sec^2 kx \, dx = \frac{1}{k} \tan kx + c$$

**Example 10.4.1**

1.  $\int \cos(2x + 1) \, dx$

**Solution:**

$$\int \cos(2x + 1) \, dx = \frac{1}{2} \times \sin(2x + 1) + c = \frac{\sin(2x + 1)}{2} + c$$

2.  $\int \sec^2(3x + 1) \, dx$

**Solution:**

$$\int \sec^2(3x + 1) \, dx = \frac{1}{3} \times \tan(3x + 1) + c = \frac{\tan(3x + 1)}{3} + c$$

3.  $\int \sin 5x \, dx$

**Solution:**

$$\int \sin 5x \, dx = -\frac{1}{5} \times \cos 5x + c$$

**10.5 Practical Exam Questions****Exercise 10.5.1 Find the following indefinite integrals:**

1.  $\int \frac{3}{(x-6)^2} dx$

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2.  $\int \frac{1}{x-5} dx$

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3.  $\int (3x^2 + \cos 2x) dx$

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4.  $\int 2 + e^{3x} dx$

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5.  $\int \sec^2 5x dx$

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**Exercise 10.5.2 Find the following indefinite integrals:**

1.  $\int \frac{6x^2}{x^3-2} dx$

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2.  $\int \frac{x}{x^2+3} dx$

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3.  $\int \frac{x}{x^2-5} dx$

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4.  $\int (4x^3 - 6x + 8) dx$

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5.  $\int (x^2 + \sqrt{x}) dx$

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**10.6 Miscellaneous exercise****Exercise 10.6.1** Find the following indefinite integrals:

1.  $\int (2x + 5)^2 dx$

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2.  $\int \frac{x^5 - 3x^6}{x^3} dx$

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3.  $\int (2 - 4x)^5 dx$

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4.  $\int \left(\frac{1}{x^3} - \frac{1}{x^4}\right) dx$

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5.  $\int \sqrt[3]{x^2} dx$

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**Exercise 10.6.2**

1.  $\int \frac{1}{x\sqrt{x}} dx$

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2.  $\int \frac{1}{(2x-4)^2} dx$

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3.  $\int \sqrt{x}(2\sqrt{x} - x) dx$

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4.  $\int (2\sqrt{x} - 1)^2 dx$

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5.  $\int \sqrt[3]{3x - 4} dx$

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