Year 11 Math Homework

Student Name: _____	**Grade**: _____
Date: _____	**Score**: _____

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1 Topic 1 — Basic Arithmetic

1.1 Fractions, Decimals and Percentages

To convert a fraction to a decimal, we divide the numerator by the denominator.

Exercise 1.1.1

1. *What is the reciprocal of:*

   (a) 9

   (b) 56

   (c) 49.5

2. *Convert the following fractions to decimals:*

   (a) $\frac{3}{8}$

   (b) $\frac{13}{25}$

   (c) $\frac{7}{40}$

3. *Express each of the following decimals as a fraction:*

   (a) 0.125

   (b) 0.025

   (c) 2.65

4. *Express each of the following percentages as a decimal:*

   (a) 65%

   (b) 115%

   (c) $12\frac{2}{5}\%$

5. *Express each of the following decimals as a percentage:*

   (a) 0.23

   (b) 1.82

   (c) 0.875

   (d) 0.0125

Exercise 1.1.2

1. *What is the reciprocal of:*

   (a) 4

   (b) 7.4

   (c) 38

   (d) 25.2

2. *Convert the following fractions to decimals:*

   (a) $\frac{5}{8}$

   (b) $\frac{13}{40}$

   (c) $\frac{8}{5}$

   (d) $\frac{17}{4}$

3. *Express each of the following decimals as a common fraction:*

   (a) 0.125

   (b) 0.025

   (c) 2.65

   (d) 0.375

4. *Express each of the following percentages as a decimal:*

   (a) 15%

   (b) 32.5%

   (c) $112\frac{1}{2}\%$

   (d) $14\frac{3}{5}\%$

5. *Express each of the following decimals as a percentage:*

   (a) 0.35

   (b) 1.36

   (c) 0.325

   (d) 0.6125

1.2 Recurring Decimals

- All common fractions $\frac{a}{b}$ where **a** and **b** are whole numbers are called rational numbers.
- Recurring decimals are *rational numbers.*
- Non-terminating and non-recurring decimals are *irrational numbers.*

Exercise 1.2.1 Express the following recurring decimals as common fractions:

1. $0.\dot{7}$

2. $0.\dot{5}\dot{3}$

3. $0.4\dot{6}$

4. $0.4\dot{5}\dot{3}$

Exercise 1.2.2 Express the following recurring decimals as common fractions:

1. $0.\dot{3}$

2. $0.\dot{4}\dot{3}$

3. $0.2\dot{6}$

4. $0.2\dot{5}\dot{3}$

5. $0.72\dot{6}$

1.3 Lowest Common Multiple

Exercise 1.3.1 Write the L.C.M of the following:

1. *5, 6, and 8*

2. *7, 14, 21 and 28*

3. *3, 7, 14 and 21*

4. *4, 12, 20 and 28*

Exercise 1.3.2

1. *The cold water tap of a bath tub can fill the tub in 10 minutes. The drain, when opened, can empty the full tub in 12 minutes. Suppose the tub is empty and the tap and drain are both opened at the same time. How long will it take to fill the tub?*

2. *Two horses run around a circular track* $300\,m$ *long. One horse runs at a steady rate of* 12 *metres per second, the other at a steady rate of* 15 *metres per second. Suppose they start at the same point and time. What is the least number of seconds that will elapse before they are again together at the starting point?*

1.4 Equivalent Fractions

- Two or more fractions that have the same value are equivalent fractions.
- If we multiply the numerator and denominator of a fraction by the same number, the value of the fraction remains the same.

Exercise 1.4.1 Complete the following equivalent fractions:

1. $\frac{3}{4} = \frac{}{12} = \frac{15}{} = \frac{}{40} = \frac{21}{}$
2. $\frac{12}{18} = \frac{2}{} = \frac{84}{} = \frac{}{9} = \frac{54}{}$
3. $\frac{6}{7} = \frac{}{35} = \frac{36}{} = \frac{}{77} = \frac{54}{}$
4. $\frac{7}{12} = \frac{}{36} = \frac{28}{} = \frac{}{96} = \frac{49}{}$

Exercise 1.4.2 Rearrange the following fractions in descending order:

1. $\frac{3}{4}, \frac{4}{5}\ and\ \frac{18}{25}$

2. $\frac{13}{15}, \frac{17}{20}\ and\ \frac{9}{10}$

3. $\frac{2}{5}, \frac{7}{15}, \frac{4}{9}\ and\ \frac{19}{45}$

4. $\frac{1}{2}, \frac{17}{32}, \frac{7}{16}\ and\ \frac{31}{64}$

1.5 Addition and Subtraction of Fractions

To add or subtract fractions:

- Find the L.C.M. of the denominators.
- Express each fraction with said L.C.M. as denominator.
- Add and/or subtract the numerators.

Exercise 1.5.1

1. $\frac{2}{5} + \frac{4}{9} - \frac{7}{15}$

2. $\frac{5}{8} - \frac{3}{16} - \frac{3}{4}$

3. $\frac{11}{12} - \frac{5}{6} + \frac{2}{9}$

4. $\frac{7}{10} + 2\frac{2}{3} - 1\frac{2}{5}$

5. $5\frac{2}{15} - 2\frac{3}{30} - 1\frac{7}{20}$

1.6 Multiplication and Division of Fractions

Exercise 1.6.1 Simplify the following:

1. $\frac{2}{3} \times \frac{6}{7} \times \frac{14}{15}$

2. $\frac{2}{3} \div \frac{8}{5} \times \frac{4}{5}$

3. $\frac{5}{12} \div \frac{3}{10} \, of \, \frac{4}{15}$

4. $4\frac{2}{7} \times 2\frac{1}{6} \times \frac{3}{16}$

5. $2\frac{2}{5} \div 1\frac{1}{3} \times \frac{5}{6}$

6. $\frac{5}{12} \div \frac{3}{10} \div 3\frac{3}{4}$

1.7 Order of Operations

Exercise 1.7.1

1. $\frac{3}{4}\,of\,16 + 5 \times (12 - 8)$

2. $\frac{2}{3} \div \frac{5}{12} + \frac{5}{6} - \frac{1}{4}\,of\,\frac{4}{5}$

3. $\frac{\frac{2}{3}+\frac{3}{4}}{1-\frac{2}{5}\times\frac{1}{3}}$

4. $6 \times (3 + 8) \times 5 - 24 \div 8$

5. $2\frac{1}{3} - \frac{2}{7}\,of\,\frac{7}{9}$

6. $\frac{\frac{5}{6}-\frac{2}{5}}{1+\frac{5}{6}\times\frac{2}{3}}$

1.8 Additions and Subtraction of Decimals

Exercise 1.8.1 Evaluate the following without the use of a calculator:

1. $8 - 3.7 + 2.5$

2. $4.6 + 3.45 + 0.375$

3. $3.65 + 0.365 + 0.0365$

4. $0.7 - 0.56 - 0.056$

5. $0.95 + 1.2 - 1.0026$

1.9 Multiplication and Division of Decimals

Exercise 1.9.1 Evaluate the following without the use of a calculator:

1. $8.264 \div 0.002$

2. 1.25×0.25

3. $\frac{0.005}{2}$

4. $\frac{1.5 \times 2.25}{0.5 \times 3.75}$

5. $\frac{4.8 \times 7.2}{0.2 \times 0.1 \times 1.8}$

1.10 Scientific Notation (Standard Form)

In scientific notation, a number is written as a number between 1 and 10 and multiplied by a power of 10. e.g. $12300 = 1.23 \times 10^4$.

Exercise 1.10.1 Express the following in scientific notation:

1. 748000
2. $8 \times 10^4 \times 3 \times 10^{-2}$
3. 423.64
4. 0.007652
5. $\frac{1.2 \times 10^5}{3 \times 10^{-3}}$
6. *12 thousand*

Exercise 1.10.2 Express the following in standard form:

1. 0.00001
2. 0.0008005
3. 0.065×10^{-5}
4. *The product of* 2.7×10^6 *and* 5×10^{-2}
5. 7.5×10^{-8} *divided by*150000
6. *The value of* ab^n *where* $a = 1.2 \times 10^{-3}$, $b = 8 \times 10^{-5}$ *and* $n = 2$

1.11 Significant Figures and Decimal Places

A significant figure is a number that is correct within some stated degree of accuracy.
The rule for significant figures are:

- All non-zero digits are significant.
- Zeros between non-zero digits are significant.
- Zeros at the end of a decimal are significant.
- Zeros before the first non-zero digit in a decimal are not significant.
- Zeros after the last non-zero digit in a whole number may or may not be significant.

Example 1.11.1 State the number of significant figures in each of these numbers:

a. 2.008

Solution: *In 2.008, the two non-zero digits are significant and two zeros between these digits are also significant. ∴ the number has four significant figures.*

b. 1028.50

Solution: *In 1028.50, the three non-zero digits are significant and both the zero in between and at the end of the decimal are significant. ∴ The number has six significant figures.*

c. 0.00125

Solution: *In 0.00125, the three non-zero digits are significant; however, the three zeros at the beginning of the decimal are not significant. ∴ the number has only three significant figures.*

d. 8000

Solution: *In 8000, the non-zero digit is significant. Either some, all or none of the final zeros could possibly be significant. If we knew that the number had been rounded off correct to:*

I *1 significant figure, then only the 8 would be significant.*

II *2 significant figures, then only the 8 and the first zero would be significant.*

III *3 significant figures, then only the 8 and the first two zeros would be significant.*

IV *4 significant figures, then all the digits would be significant.*

Exercise 1.11.1 How many significant figures are there in each of the following?

1. 50000

2. 25000

3. 12500

4. 12580

5. 0.125

6. 0.05

7. 0.0025

8. 25

9. 25.00

Exercise 1.11.2 Round off the following to the number of significant figures indicated in brackets.

1. $4358\,(3)$

2. $3.294\,(2)$

3. $15.066\,(4)$

4. $0.0072\,(1)$

5. $2.048 \times 10^3\,(2)$

6. $8.585\,(2)$

7. $0.9058\,(3)$

8. $0.00603\,(4)$

1.12 Approximations

Exercise 1.12.1 By expressing each number correct to one significant figure, find approximately the value of the following:

1. 29.4×0.62
2. $4125 \div 21.5$
3. $\frac{715 \times 18}{0.042}$
4. 48.2×13.9
5. $\frac{56.3}{0.0063}$

Exercise 1.12.2 Express each of the following correct to one significant figure:

1. 415.3
2. 0.0205
3. 0.695
4. 33.85
5. 191.5

Exercise 1.12.3 By expressing each number correct to one significant figure, find approximately the value of the following:

1. 0.025×0.052
2. $\frac{72.5}{4.9}$
3. 98.3×1.005
4. $\frac{16 \times 61}{3.1}$
5. $\frac{118.5 \times 0.002}{0.048}$

1.13 Powers and Roots

Exercise 1.13.1 Evaluate the following:

1. $(\frac{3}{4})^2$
2. $(3\frac{1}{3})^2$
3. $(0.01)^2$
4. $(0.007)^2$
5. 3.2^2
6. $(0.4)^3$
7. $(\frac{3}{4})^3$
8. $\sqrt{5^2+12^2}$

Exercise 1.13.2 Without the aid of a calculator, find the exact value of the following:

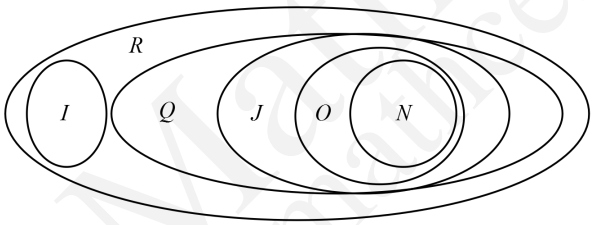
1. $(0.5)^3$
2. $(3\frac{1}{2})^3$
3. 0.002^3
4. $\sqrt[3]{\frac{8}{125}}$
5. $\sqrt[3]{0.216}$
6. $\sqrt{(1\frac{1}{2})^2+2^2}$

1.14 Irrational Numbers

- A rational number is one that can be expressed in the form $\frac{a}{b}$ where **a** and **b** are integers and $b \neq 0$.
- There are numbers which when expressed as decimals neither terminate nor recur. These are called irrational numbers.
- Irrational numbers such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and $\sqrt{7}$ are called surds.

1.15 Real Numbers

- The entire collection of rational and irrational numbers is called the set of real numbers.
- The following Venn diagram shows the various subsets of the set of real numbers.



- The various subsets are represented by capital letters:
  - R = real numbers
  - I = irrational numbers
  - Q = rational numbers
  - J = integers
  - O = whole numbers
  - N = natural numbers

1.16 Operations with Surds

- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \sqrt{a} \div \sqrt{b}$ or $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ or $\sqrt{\frac{a}{b}} = \frac{\sqrt{ab}}{b}$
- $(\sqrt{a})^2 = a$

Exercise 1.16.1 Simplify the following:

1. $\sqrt{27}+\sqrt{108}-\sqrt{12}$
2. $3\sqrt{125}-2\sqrt{80}$
3. $5\sqrt{3}+\sqrt{20}-2\sqrt{12}+\sqrt{45}$
4. $\sqrt{x^3}+\sqrt{x}-\sqrt{4x}$
5. $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$
6. $(\sqrt{3}+\sqrt{5})^2$

Exercise 1.16.2

1. $2\sqrt{7}\times\sqrt{63}$
2. $\sqrt{27}\times\frac{1}{3}\sqrt{3}$
3. $\sqrt{8}\times\sqrt{5}\times\sqrt{125}$
4. $(5-\sqrt{2})(5+\sqrt{2})$
5. $(2\sqrt{3}-1)(2\sqrt{3}+1)$

Exercise 1.16.3 Simplify the following by rationalising the denominator in each case:

1. $\frac{3}{\sqrt{2}}$
2. $\frac{\sqrt{7}}{\sqrt{14}}$
3. $\frac{4\sqrt{2}}{3\sqrt{8}}$
4. $\frac{3\sqrt{2}+\sqrt{5}}{2\sqrt{5}}$
5. $\frac{4\sqrt{3}-2\sqrt{2}}{3\sqrt{2}}$

Exercise 1.16.4 Express each of the following in simplest surd form:

1. $4\sqrt{44}$

2. $\frac{2}{3}\sqrt{243}$

3. $5\sqrt{125}$

4. $\frac{1}{5}\sqrt{175}$

5. $3\sqrt{28}$

Exercise 1.16.5 Simplify the following:

1. $\sqrt{27}-\sqrt{18}+\sqrt{3}$

2. $\sqrt{125}-5\sqrt{2}+\sqrt{50}$

3. $5\sqrt{3}+\sqrt{27}-\sqrt{45}$

4. $\sqrt{98}-\sqrt{12}-2\sqrt{20}$

5. $8\sqrt{2}+\sqrt{12}-3\sqrt{3}-3\sqrt{8}$

Exercise 1.16.6 Simplify the following:

1. $3\sqrt{3}\times 2\sqrt{2}$

2. $4\sqrt{3}\times\sqrt{18}$

3. $3\sqrt{12}\times 5\sqrt{2}$

4. $5\sqrt{12}\times\sqrt{27}$

5. $4\sqrt{5}\times\sqrt{20}$

Exercise 1.16.7 Simplify the following:

1. $\frac{\sqrt{8}}{\sqrt{2}}$

2. $\frac{2}{\sqrt{3}}$

3. $\frac{\sqrt{27}}{\sqrt{3}}$

4. $\frac{3\sqrt{2}}{\sqrt{6}}$

5. $\frac{\sqrt{60}}{\sqrt{5}}$

Exercise 1.16.8 Expand each of the following, expressing the result in simplest surd form.

1. $(\sqrt{5}+\sqrt{3})(\sqrt{7}-\sqrt{2})$

2. $(2\sqrt{3}-3)(2\sqrt{3}+5)$

3. $(2\sqrt{2}-\sqrt{6})(2\sqrt{3}-1)$

4. $(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3})$

5. $(2\sqrt{7}-\sqrt{2})^2$

Exercise 1.16.9 Express the following fractions with a rational denominator.

1. $\frac{1}{\sqrt{5}-\sqrt{3}}$

2. $\frac{1}{2\sqrt{3}+\sqrt{2}}$

3. $\frac{3\sqrt{6}}{2\sqrt{2}+\sqrt{3}}$

4. $\frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}+\sqrt{2}}$

Exercise 1.16.10

1. *A light flashes every 3 minutes, a second light flashes every 4 minutes, and third light flashes every 5 minutes. Suppose all three light flash together at noon. What is the next time of the day they will all flash together?*

2. *If 32 is added to one-third of a number, the triple of the number is the result. What is the number?*

3. *The symbol* $5!$ *means* $5 \times 4 \times 3 \times 2 \times 1 = 120$. *Suppose that N! ends in exactly 3 zeros after fully multiplying out. What is the smallest value that N can be?*

4. *The five-digit number 326A8 is divisible by 12. What digit/s does A represent?*

5. *The sum of 7 consecutive odd numbers is 161. What is the sum of the smallest and largest of these numbers?*