Year 11 Math Homework

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

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2 Year 11 Topic 2 — The Language of Sets

2.1 Describing Sets

- A set is a defined collection of objects.
- These objects are known as *members.*
- Sets can be described by listing the members inside curly brackets:
  - $S = \{2, 4, 6, 8, 10\}$
  - Read as *'S is the set whose members are 2, 4, 6, 8, 10'.*
- Alteratively it can be done by writing a description such as:
  - T = { even integers from 0 to 10}
  - Read as *'T is the set of even integers from 0 to 10'.*

2.2 Equal Sets

- If two sets have exactly the same members, they are equal.
- The order in which the members are written doesn't matter, neither does repetition.
- For example: $\{a, b, c, d\} = \{a, d, c, b, a\} = \{b, c, a, d, a, c, b\}$

2.3 Members and Non-members

- The symbol $\in$ means *'is a member of'.*
- The symbol $\notin$ means *'is not a member of'.*
- For example, if $B = \{a, b, c, d, e\}$, then $a \in B$ and $c \in B$ but $f \notin B$.

2.4 The Size of a Set

- A set may be finite or infinite.
- If a set S is finite, then |S| is the symbol for the number of members in S.
- For example: $|\{a, e, i, o, u\}| = 5$
- Keep in mind: $5 \in \{5\}$ and $5 \neq |\{5\}|$ and $|\{5\}| = 1$.

2.5 The Empty Set

- The symbol $\emptyset$ represents the empty set.
- The empty set is finite and its number of members is zero.
  - ie; $|\emptyset| = 0$

Exercise 2.5.1 State whether each set is finite/infinite. If it is finite, state the number of members:

1. $\{2, 4, 6, 8, ...\}$ ______
2. $\{0, 1, 2, 3, ...9\}$ ______
3. $\{a, b, e, f, g, i, p, a, e\}$ ______
4. $\{multiples\,of\,5\,that\,are\,less\,than\,100\}$ ______
5. $\{n : \, n\,is\,a\,positive\,integer\,and\,1 < n < 25\}$ ______

2.6 Subsets of Sets

- A set of B is called a subset of a set of C if every member of B is a member of C.
- This notation is written as $B \subset C$.
- The symbol $\not\subset$ means *'not a subset of'*.
- For example: $\{children\,in\,Australia\} \subset \{people\,in\,Australia\}$ but $\{a, b, c\} \not\subset \{a, b, d, e\}$

Exercise 2.6.1 State whether each of the following statements is true or false:

1. *If* $A = \{0, 0\}$, *then* $|A| = 1$. ______
2. $|\{0\}| = 0$. ______
3. $|\{1, 1\}| = 1$. ______
4. *If two sets have the same number of members, then they are equal.* ______
5. *If two sets are equal, then they have the same number of members.* ______
6. *If* $A \subset B$ *and* $B \subset A$, *then* $A = B$. ______
7. *If* $A \subset B$ *and* $B \subset C$, *then* $A \subset C$. ______
8. $|\{20, 21, 22, 23, ...40\}| = 20$. ______

2.7 Unions and Intersections

- The union A∪B of two sets A and B is the set of everything belonging to A, B or both.
- The intersection A∩B is the set of everything belonging to both A and B.
- For example, if $A = \{a, b, c, d, e\}$ and $B = \{a, c, f\}$,
  then $A \cup B = \{a, b, c, d, e, f\}$ and $A \cap B = \{a, c\}$.
- Two sets A and B are called disjointed if they have no members in common, ie; A∩B = $\emptyset$.
- In short, '**Or**' means Union, '**And**' means Intersection.

2.8 The Universal Set and the Complement of a Set

- A universal set is the set of everything under discussion in a particular situation.
- Once a universal set E is fixed, then the complement $\bar{A}$ of any set A is the set of all members of that universal set which are not in A.
- For example: If $A = \{2, 4, 6, 8, 10\}$ and $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
  then $\bar{A} = \{1, 3, 5, 7, 9\}$
- Notice that: $A \cup \bar{A} = E$ and $A \cap \bar{A} = \emptyset$
- '**Not**' means Complement: $\bar{A}$ = $\{x \in E : x\, is\, not\, a\, member\, of\, A\}$

Exercise 2.8.1 Find $A \cup B$ and $A \cap B$ for each pair of sets:

1. $A = \{m, n\}$*, and B = {m, n, o, p}.*

2. $A = \{c, o, m, p, u, t, e, r\}$ *and* $B = \{s,\ o,\ f,\ t,\ w,\ a,\ r,\ e\}$

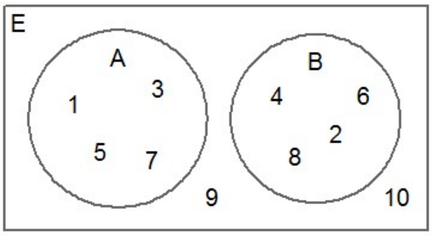
3. $A = \{1, 2, 3, 4, 6, 8\}$ *and* $B = \{2, 3, 4, 6, 8, 9\}$

4. *A = {prime numbers less than 15} and B = {odd numbers less than 15}*

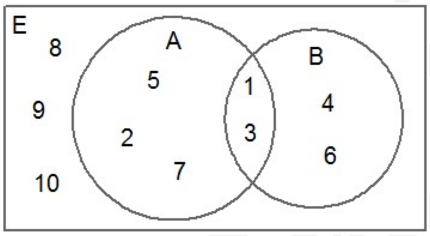
2.9 Venn Diagrams

- A Venn diagram is a diagram used to represent the relationship between sets.
- For example, if the universal set is E = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10:

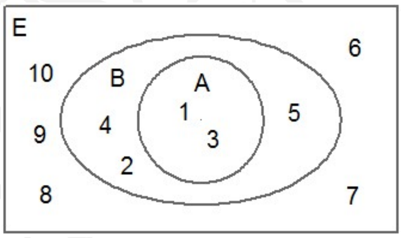
a. $A = \{1, 3, 5, 7\}$ and $B = \{2, 4, 6, 8, \}$



b. $A = \{1, 2, 3, 5, 7\}$ and $B = \{1, 3, 4, 6\}$



c. $A = \{1, 3\}$ and $B = \{1, 2, 3, 4, 5\}$



- Compound sets of $A\cup B$, $A\cap B$ and $\bar{A}\cap B$ can be visualised by shading regions of a Venn diagram.

2.10 The Counting Rule for Sets

- The size of the union $A \cup B$ is not equal to the sum of sizes of A and B.
- This is because the members of the intersection $A \cap B$ would be counted twice.
- Hence $|A \cap B|$ needs to be subtracted as shown below:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Exercise 2.10.1 Suppose $A = \{1, 3, 5, 7\}$ and $B = \{3, 4, 5, 8, 10\}$ with universal set {1, 2, . . . 10}. List the members of:

1. $\bar{A}$ ________________________________

2. $\bar{B}$ ________________________________

3. $A \cap B$ ________________________________

4. $\bar{A} \cup \bar{B}$ ________________________________

5. $\overline{A \cup B}$ ________________________________

6. $\bar{A} \cap \bar{B}$ ________________________________

7. $\overline{A \cap B}$ ________________________________

Exercise 2.10.2 Given the following sets: $A = \{a, b, c, d, e, f, \}$, $B = \{b, c, d\}$, $C = \{d, e, f, g, h\}$ and $D = \{g, h, i\}$. Complete the following:

1. $n(A) =$ ________________________________

2. $B \subset$ ________________________________

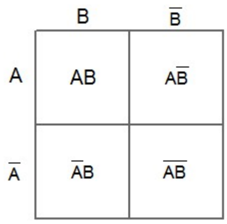
3. $A \cap C =$ ________________________________

4. $C \cup D =$ ________________________________

5. *List the subsets of B* ________________________________

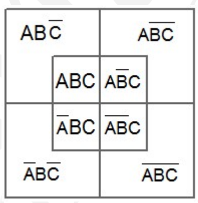
2.11 Lewis Carroll's Bilateral Diagrams

- A and $\bar{A}$ are represented by horizontal rows of cells.
- B and $\bar{B}$ are related vertical cell arrangement.
- For example:



2.12 Lewis Carroll's Trilateral Diagrams

- A and $\bar{A}$ are represented by horizontal rows of cells.
- B and $\bar{B}$ are related vertical cell arrangement.
- C is related to the inside of the middle square (Inner Cells).
- $\bar{C}$ are represented by cells outside of the middle square (Outer Cells).
- For example:



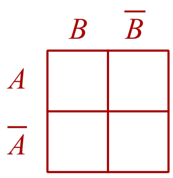
Exercise 2.12.1 Use Lewis Carroll diagrams graph the following:

a. $A \cup \bar{B}$

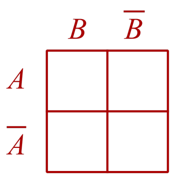
b. $\bar{A} \cup B$

c. $A \cap \bar{B}$

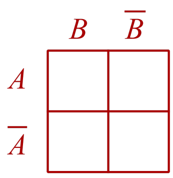
d. $\overline{A \cap B}$



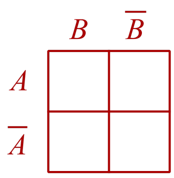
a



b



c



d

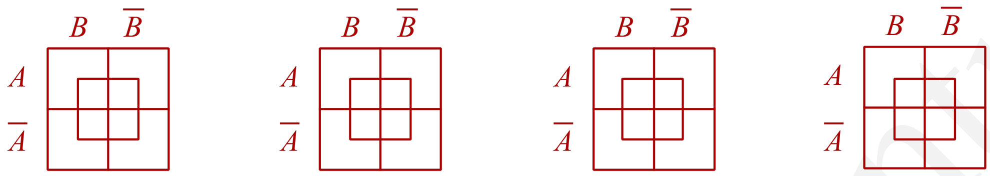
Exercise 2.12.2 Use Lewis Carroll diagrams graph the following:

a. $A \cap B \cap C$

b. $A \cap \bar{B} \cap C$

c. $\bar{A} \cap \bar{B} \cap C$

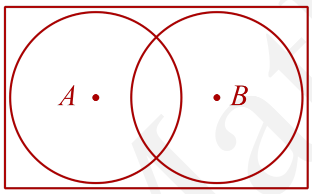
d. $A \cup (\overline{B \cap C})$

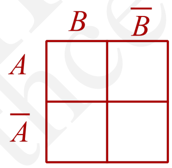


a b c d

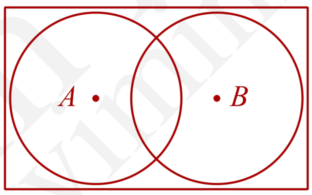
Exercise 2.12.3 Use Venn diagram and Lewis Carroll diagrams to graph the following:

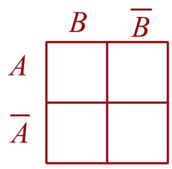
1. $A \cup \bar{B}$





2. $A \cap \bar{B}$





3. $A \cap (\overline{B \cup C})$