Year 11 Math Homework

Student Name: ______________	**Grade**: ______________
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4 Year 11 Topic 4 — Numbers and Functions (Part 1)

4.1 Numbers and Functions (Revision)

4.1.1 Surds and their Arithmetic

Exercise 4.1.1

1. *Use the result* $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ *to simplify these fractions:*

   (a) $\frac{\sqrt{72}}{\sqrt{98}}$

   (b) $\frac{\sqrt{50}}{\sqrt{8}}$

   (c) $\frac{\sqrt{52}}{\sqrt{91}}$

   (d) $\frac{\sqrt{175}}{\sqrt{28}}$

2. *Simplify each of these expressions completely:*

   (a) $\sqrt{96} - \sqrt{24} - \sqrt{54}$

   (b) $\sqrt{45} + \sqrt{80} - \sqrt{125}$

   (c) $\sqrt{63} + \sqrt{72} - \sqrt{50}$

   (d) $\sqrt{20} - \sqrt{12} + \sqrt{108}$

3. *Expand the following expressions and simplify them:*

   (a) $(\sqrt{3} - 1)(\sqrt{2} - 1)$

   (b) $(\sqrt{a} - 1)(\sqrt{a} + 1)$

   (c) $(2\sqrt{5} + \sqrt{3})(2 - \sqrt{3})$

   (d) $(\sqrt{x+1} + \sqrt{x-2})^2$

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Exercise 4.1.2 Rationalising the denominator

1. *Fully simplify these fractions:*

   (a) $\frac{6\sqrt{3}\times5\sqrt{2}}{\sqrt{12}\times\sqrt{18}}$

   (b) $\frac{5\sqrt{44}\times\sqrt{14}}{\sqrt{24}\times3\sqrt{33}}$

2. *Simplify the following by rationalising the denominator of each fraction:*

   (a) $\frac{1}{3+\sqrt{6}} - \frac{2}{\sqrt{6}}$

   (b) $\frac{1}{3\sqrt{2}+1} + \frac{1}{1-3\sqrt{2}}$

3. *Determine, without using a calculator, which is the greater number in each pair:*

   (a) $15 - 7\sqrt{2}$ *or* $3 + 2\sqrt{2}$

   (b) $2\sqrt{6} - 3$ *or* $7 - 2\sqrt{6}$

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4.1.2 Equality of Surdic Expressions

Exercise 4.1.3

1. *Find the value of integers x, y and z, given that z has no squares as factors:*

   (a) $x + y\sqrt{3} = (6 + \sqrt{3})^2$

   (b) $x + y\sqrt{z} = (3 + \sqrt{5})^2$

2. *Find the rational numbers a and b such that:*

   (a) $a + b\sqrt{3} = \frac{1}{2-\sqrt{3}}$

   (b) $a + b\sqrt{6} = \frac{2\sqrt{6}+1}{2\sqrt{6}-3}$

3. *Find the rational value of a and b, with $a > 0$ by forming two simultaneous equations and solving them:* $(a + b\sqrt{2})^2 = 3 + 2\sqrt{2}$

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4.1.3 Relations and Functions

- A function is a set of ordered pairs in which no two ordered pairs have the same x-coordinate.
- The domain of a function is the set of all x-coordinates of the ordered pairs.
- The range of a function is the set of all y-coordinates.

Exercise 4.1.4

1. *Given that* $f(x) = x^3 - x + 1$, *evaluate and simplify the following:*

   (a) $\frac{f(h)-f(0)}{h}$

   (b) $\frac{1}{6}\left(f(0) + 4f(\frac{1}{2}) + f(1)\right)$

2. *Find the natural domains of the following:*

   (a) $f(x) = \sqrt{9 - x^2}$

   (b) $f(x) = \frac{1}{x^2-5x+6}$

   (c) $g(x) = \frac{x-3}{x^2-9}$

3. *If* $f(x) = \frac{1}{1-x}$, *find f(a-b).*

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Exercise 4.1.5

1. *If* $f(x) = 3^x$, *show that* $f(-x) = \frac{1}{f(x)}$

2. *If* $h(x) = \frac{x}{x^2-1}$, *show that* $h(\frac{1}{x}) = -h(x)$ *for* $x \neq 0$

3. *If* $f(x) = x + \frac{1}{x}$, *show that* $f(x) \times f(x + \frac{1}{x}) = f(x^2) + 3$

4. *Given the functions* $f(x) = x^2$, $F(x) = x - 3$, $g(x) = 2^x$ *and* $G(x) = 3x$, *find:*

   (a) $F\left(f(x)\right)$

   (b) $G\left(g(x)\right)$

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4.1.4 Inverse Relations and Functions

- The inverse relation is obtained by reversing the values of each ordered pair.
- The domain of the inverse is the range of the relation and the range of the inverse is the domain of the relation.
- The graph of the inverse relation is obtained by reflecting the original graph in the line $y = x$.
- To find the equations and conditions of the inverse relation, write x for y, y for x and then solve for y.
- The inverse relation of a given relation is a function if and only if no horizontal line crosses the original graph more than once.

Exercise 4.1.6 Find the inverse algebraically by swapping x and y and then making y the subject:

1. $y = \frac{1}{x-1}$

2. $y = \frac{x+3}{x-3}$

3. $y = \frac{2x}{x+3}$

4. $y = \frac{2x-2}{x-2}$

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Exercise 4.1.7

1. *Each pair of functions f(x) and g(x) are mutually inverse. Verify in each case by substitution that:* ***(i)*** $f(g(2)) = 2$ *and* ***(ii)*** $g(f(2)) = 2$*:*

   (a) $f(x) = x + 13$ *and* $g(x) = x - 13$

   (b) $f(x) = x^3 - 6$ *and* $g(x) = \sqrt[3]{x + 6}$

2. *Show that the inverse function of* $y = \frac{ax+b}{x+c}$ *is* $y = \frac{b-cx}{x-a}$*, for* $x \leq 0$.

3. *Hence show that* $y = \frac{ax+b}{x+c}$ *is its own inverse if and only if* $a + c = 0$.

Exercise 4.1.8 Express $1 - \frac{2}{1-x}$ **as a single fraction and hence find its reciprocal.**

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Exercise 4.1.9

1. *If* $f(x) = \frac{1}{1-x}$, *find* $f(f(x))$.

2. *Find the range of the function* $f(x) = \frac{1}{x^2+4x+7}$.

3. *Solve* $\frac{x}{1-x} > \frac{1}{3}$.

4. *Solve* $\frac{1}{1-x^2} \leq 4$.

5. *Solve* $|1+3x| = x-2$.

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Exercise 4.1.10

1. *Show that* $\frac{x^2+1}{x^2+4} = 1 - \frac{3}{x^2+4}$

2. *Simplify* $\frac{3^n+3^{n-2}}{3^{n-1}}$.

3. *The equation* $\frac{1}{1+x^2} = k$ *has two distinct real roots. Find the possible values of k.*

4. *Prove that* $\frac{1}{5-\sqrt{3}} + \frac{1}{5+\sqrt{3}}$ *is rational.*

5. *Solve* $x + |x| = 4$

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