Year 9 Term 1 Homework Worked Solutions

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

Table of contents

10 Year 9 Term 1 Week 10 Homework Solutions . . . **1**
10.1 Chapter Review . . . 1
10.1.1 Significant figures . . . 1
10.1.2 Recurring decimals . . . 2
10.1.3 Rates . . . 2
10.1.4 Algebra . . . 3
10.1.5 Consumer arithmetic . . . 5
10.1.6 Equations, inequations and formulae . . . 6
10.2 Maths challenge . . . 7
10.3 Miscellaneous exercise . . . 8
10.4 Practical Exam Questions . . . 9

This edition was printed on March 15, 2022.

Camera ready copy was prepared with the LaTeX2e typesetting system.

10 Year 9 Term 1 Week 10 Homework Solutions

10.1 Chapter Review

10.1.1 Significant figures

A significant figure is a number that is correct within some stated degree of accuracy. The rule for significant figures are:

- All non-zero digits are significant.
- Zeros between non-zero digits are significant.
- Zeros at the end of a decimal are significant.
- Zeros before the first non-zero digit in a decimal are not significant.
- Zeros after the last non-zero digit in a whole number may or may not be significant.

Exercise 10.1.1 Round off 76.543 correct to:

1. *2 significant figures* [*Answer:* 77].
2. *3 significant figures* [*Answer:* 76.5].
3. *4 significant figures* [*Answer:* 76.54].

Exercise 10.1.2 State the number of significant figures in each of the following:

1. *8004* [*Answer:* 4 *significant figures*].
2. *0.36* [*Answer:* 2 *significant figures*].
3. *18.020* [*Answer:* 5 *significant figures*].
4. *0.01201200* [*Answer:* 7 *significant figures*].

Exercise 10.1.3 Round off each of the following correct to 3 significant figures:

1. *8280* [*Answer:* 8280].
2. *364005* [*Answer:* $364,000$].
3. *0.25949* [*Answer:* 0.259].
4. *194.62* [*Answer:* 195].

10.1.2 Recurring decimals

Exercise 10.1.4 Convert each of these recurring decimals to a fraction or a mixed numeral, in simplest form:

1. $0.\dot{2}\dot{7}$

Solution: $\text{Let } x = 0.\dot{2}\dot{7} \Rightarrow 100x = 27.\dot{2}\dot{7} \Rightarrow 100x - x = 27.\dot{2}\dot{7} - 0.\dot{2}\dot{7}$

$99x = 27 \Rightarrow \therefore x = \dfrac{27}{99} = \dfrac{3}{11}.$

2. $0.41\dot{6}$

Solution: $\text{Let } x = 0.41\dot{6} \Rightarrow 1000x = 416.\dot{6} \text{ and } 100x = 41.\dot{6}$

$1000x - 100x = 416.\dot{6} - 41.\dot{6} \Rightarrow 900x = 375 \Rightarrow \therefore x = \dfrac{375}{900} = \dfrac{5}{12}.$

10.1.3 Rates

- A rate is a comparison of two unlike quantities.
- A rate is a measure of how one quantity is changing with respect to another.
- To be in simplest form, a rate must be expressed as a quantity per one unit of another quantity.

Exercise 10.1.5 Complete the following equivalent rates:

1. *60 m/s =* [*Answer:* 216]. *km/h*
2. *1.5 m/min =* [*Answer:* 2.16]. *km/day*
3. *25 mL/s =* [*Answer:* 90]. *L/h*
4. *1.25 t/h =* [*Answer:* 30,000]. *kg/day*

Exercise 10.1.6 Further applications

1. *Calculate the daily interest rate on a credit card if the annual rate is 18.5% p.a.*

Solution: $\dfrac{18.5}{365} \times 100\% = 0.0507\% \text{ daily.}$

2. *Convert $734.50/quarter to an equivalent weekly rate.*

Solution: $52 \div 4 = 13 \text{ weeks/quarter} \Rightarrow 734.5 \div 13 = \$56.50/\text{week.}$

10.1.4 Algebra

Exercise 10.1.7 Find the value of the following expressions if $a = 3,\ b = -4\ and\ c = \frac{1}{2}$

1. $a^2(c + b)$ [*Answer:* $3^2(\frac{1}{2} - 4) = -31\frac{1}{2}$ *or* -31.5].
2. $a^2 + b^3 + c$ [*Answer:* $3^2 + (-4)^3 + \frac{1}{2} = -54.5$ *or* $-54\frac{1}{2}$].
3. $\frac{1}{c} - \frac{1}{b}$ [*Answer:* $2 + \frac{1}{4} = 2\frac{1}{4}$].

Exercise 10.1.8 Simplifying the following expressions:

1. $x^2 + 2x + 2x^2 + 3x + 3x^3 - x$ [*Answer:* $= 3x^3 + 3x^2 + 4x$].
2. $(-ab) \times (-bc) \times 2ab$ [*Answer:* $= 2a^2b^3c$].
3. $\frac{1}{4}x \times 4x^2 \times (-2x)$ [*Answer:* $-2x^4$].
4. $9xy \div 3x \times 2y$ [*Answer:* $6y^2$].
5. $\frac{3x-x+2x}{2\times 2x}$ [*Answer:* $= \frac{4x}{4x} = 1$].
6. $\frac{5a\times 4b\times 2c}{10c\times b\times 8}$ [*Answer:* $\frac{40abc}{80bc} = \frac{a}{2}$].
7. $\frac{3}{2x} - \frac{1}{3x}$

   Solution:
   $$\frac{3}{2x} - \frac{1}{3x} = \frac{3\times 3}{3\times 3x} - \frac{1\times 2}{3x\times 2} = \frac{9}{6x} - \frac{2}{6x} = \frac{7}{6x}.$$

8. $\frac{x}{3p} + \frac{3x}{4p}$

   Solution:
   $$\frac{x}{3p} + \frac{3x}{4p} = \frac{4x}{12p} + \frac{9x}{12p} = \frac{13x}{12p}.$$

9. $\frac{2x}{3a} + \frac{y}{4a}$

   Solution:
   $$\frac{2x}{3a} + \frac{y}{4a} = \frac{8x}{12a} + \frac{3y}{12a} = \frac{8x + 3y}{12a}.$$

10. $\frac{2}{x} \times \frac{x}{3} \times \frac{9x}{4}$

    Solution:
    $$\frac{2}{x} \times \frac{x}{3} \times \frac{9x}{4} = \frac{3x}{2}.$$

Exercise 10.1.9 Simplify each expression by expanding the grouping symbols and then collecting like terms.

1. $5(a+7) - 3(a+6)$

Solution: $$5(a+7) - 3(a+6) = 5a + 35 - 3a - 18 = 2a + 17.$$

2. $5(b-5) - 3(b+3)$

Solution: $$5(b-5) - 3(b+3) + 5b - 25 - 3b - 9 = 2b - 34.$$

3. $c(c-4) - 5(c-4)$

Solution: $$c(c-4) - 5(c-4) = c^2 - 4c - 5c + 20 = c^2 - 9c + 20.$$

Exercise 10.1.10 Expand and simplify each of these expressions:

1. $(2m-n)(2m+n)$

Solution: $$(2m-n)(2m+n) = 4m^2 - n^2.$$

2. $(2n+m)(n+2m)$

Solution: $$(2n+m)(n+2m) = 2n^2 + 4mn + mn + 2m^2 = 2m^2 + 5mn + 2n^2.$$

3. $(3p+2q)(2p+3q)$

Solution: $$(3p+2q)(2p+3q) = 6p^2 + 0pq + 4pq + 6q^2 = 6p^2 + 13pq + 6q^2.$$

4. $(2x+3y)^2$

Solution: $$(2x+3y)^2 = 4x^2 + 12xy + 9y^2.$$

5. $5(x-5)^2 - 4(x-4)^2 + 3(x-3)^2$

Solution:
$$\begin{aligned} 5(x-5)^2 - 4(x-4)^2 + 3(x-3)^2 &= 5(x^2 - 10x + 25) - 4(x^2 - 8x + 16) + 3(x^2 - 6x + 9) \\ &= 5x^2 - 50x + 125 - 4x^2 + 32x - 64 + 3x^2 - 18x + 27 \\ &= 4x^2 - 36x + 88. \end{aligned}$$

10.1.5 Consumer arithmetic

Exercise 10.1.11

1. *Due the the economic downturn, the employees at a small financial company have their pay reduced by 8%. Calculate the new annual pay for an employee who previously earned \$920 per week.*

Solution: *Reduce by* 8%, *paid* $92\% \Rightarrow 920 \times 92\% \times 52 = \$44,012.80$

2. *A real estate agent is paid a commission of 3% on the first \$200,000 of the value of a property and 2% of the remaining value. Find the total commission on the sale of a house sold for \$585,000.*

Solution: $3\% \times 200,000 + 2\% \times 385,000 = \$13,700.$

3. *Raymond normally earns \$712.50 for a 38-hour work. For how many hours would he have work in a week where his total pay is \$825.00, if all overtime is paid at the one and a half times rate?*

Solution: *Hour rate* $= 712.5 \div 38 = \$18.75, \Rightarrow \$18,75 \times 1.5 = \$28.125/hour.$

$\$825 - \$712.50 = 112.5 \Rightarrow \$112.5 \div 28.125 = 4\,hours.$

$\therefore$ *the total working hour:* $= 38 + 4 = 42\,hrs.$

4. *Jane purchased a LCD TV with a marked price of \$1500 at the mid-year sales, where everything in the store was advertised at 15% off. She was also given a further discount of 3% because she paid cash. How much did she pay for the TV?*

Solution: $\$1500 \times 85\% \times 97\% = \$1236.75.$

5. *A manufacturer sells an MP3 player to a wholesaler at cost plus 20%. The wholesaler then marks up the price by a further 25% and sells them to a retailer. The retailer then sells the MP3 player for \$42.00 each, making a profit of 40%. How much would it cost to manufacture 2000 MP3 players?*

Solution: $C \times 120\% \times 125\% \times 140\% = \$42.00 \Rightarrow C = \$20.00$

Total cost: $= \$20 \times 2000 = \$40,000.$

10.1.6 Equations, inequations and formulae

Exercise 10.1.12 solve the following number problems:

1. *Two-fifths of a number increased by 5 is 13. What is the number?*

Solution: $\frac{2}{5} \times n + 5 = 13 \;\Rightarrow\; 2N + 25 = 65 \;\Rightarrow\; N = 20.$

2. *Nine times a number diminished by 27 is 27. Find the number.*

Solution: $9N - 27 = 27 \;\Rightarrow\; 9N = 54 \;\Rightarrow\; N = 6.$

3. *Three tenths of a number is one more than two fifths of the number. What is the number?*

Solution: $\frac{3}{10}N = \frac{2}{5}N + 1 \;\Rightarrow\; 3N = 4N + 10 \;\Rightarrow\; N = -10.$

4. *Solve this equation* $\frac{x+3}{4x} = 5 + \frac{1}{x}$

Solution: $\frac{x+3}{4x} = \frac{20x+4}{4x} \; x + 3 = 20x + 4 \;\Rightarrow\; x = -\frac{1}{19}.$

5. *One number is three times more than another number. The sum of the large number and twice the smaller number is 12. Find the numbers.*

Solution: $\begin{cases} A = 3B \\ A + 2B = 12 \end{cases} \;\Rightarrow\; 3B + 2B = 12 \;\Rightarrow\; \begin{cases} A = 7\frac{1}{5} \\ B = 2\frac{2}{5}. \end{cases}$

6. *The difference of two numbers is 26. The large number is 8 more than ten times the small number. What are the numbers?*

Solution: $\begin{cases} A - B = 26 \\ A = 10B + 8 \end{cases} \;\Rightarrow\; 9B = 18 \;\Rightarrow\; \begin{cases} A = 28 \\ B = 2. \end{cases}$

7. *A rocket plus its fuel weighs 5200 kg. After one quarter of fuel is used, the rocket and the remaining fuel weigh 4600 kg. Find the weight of the rocket?*

Solution: $5200 - 4600 = 600\,kg = \frac{1}{4}\textit{fuel} \;\Rightarrow\; \textit{total fuel} = 4 \times 600 = 2400\,kg.$

$\therefore$ *The weight of the rocker:* $= 5200 - 2400 = 2800\,kg.$

10.2 Maths challenge

Exercise 10.2.1

1. *If x and y are non-negative integers and* $3x + 4y = 96$, *how many pairs (x, y) are there?*

   A. 6 *B. 8* *C. 10* ***D. 9*** *E. 12*

   Solution: *9 pairs;* $3x = 96 - 4y = 4(24 - y)$

   $$\begin{cases} x \text{ is a multiple of 4 and } y < 24. \\ y \text{ is a mulitple of 3 and } x < 32 \end{cases}$$

   so $x = 0, 4, 8, 12, 16, 24, 20, 28, 32,$ *and* $y = 24, 21, 18, 15, 12, 8, 6, 3$ *and* $0.$

2. *Tickets to a concert cost \$9 for an adult and \$6 for a child. If a total of 120 adults and children attended the concert and \$840 was collected, What is the difference of the number of children and the number of adults?*

   A. 40 *B. 50* *C. 60* *D. 70* *E. 80*

   Solution: *Let number of children be x, then the adult will be 120 -x.*

   $9(120 - x) + 6x = 840 \Rightarrow 1080 - 9x + 6x = 840$

   $1080 - 3x = 840 \Rightarrow 3x = 1080 - 840,$

   $\therefore x = 80 \Rightarrow$ *Adult:* $= 40 \Rightarrow$ *difference* $= 80 - 40 = 40.$

3. *When the digits of a two-digit number, neither digit zero, are reversed the number formed is 36 less than the original number. the sum of the digits of the original number could be:*

   A. 4 ***B. 6*** *C. 15* *D. 16* *E. 18*

   Solution: *Let the numbers be a and b,*

   then 10a + b - 10b -a = 9(a-b) = 36

   so the difference between digits is (a-b) = 4,

   thus the number could be 95, 84, 73, 62 and 51.

   The digit sums are 14, 12, 10, 8 and 6.

10.3 Miscellaneous exercise

Exercise 10.3.1 The following currency conversions show the value of 1 Australian dollar (AUD$1) in USD$, EURO and NZD$.

AUD$1 = USD$0.6402	*AUD$1 = 0.5054 EURO*	*AUD$1 = NZD$1.2733*

Use these currency conversions to convert:

1. *AUD$50 into USD$*   [*Answer:* $USD\$32.01$].
2. *AUD$25 into EURO*   [*Answer:* $\$12.635$].
3. *USD$1200 into NZD$*   [*Answer:* $\frac{1200}{0.6402} \times 1.2733 = \$2386.69.$].

Exercise 10.3.2 Simplifying the following expressions:

1. $\frac{5a \times 4b \times 2c}{10c \times b \times 8c}$

Solution: $$\frac{5a \times 4b \times 2c}{10c \times b \times 8c} = \frac{40abc}{80bc^2} = \frac{a}{2c}.$$

2. $\frac{8}{a} \times \frac{2a}{15} \div \frac{8}{3}$

Solution: $$\frac{8}{a} \times \frac{2a}{15} \div \frac{8}{3} = \frac{8}{a} \times \frac{2a}{15} \times \frac{3}{8} = \frac{2}{5}.$$

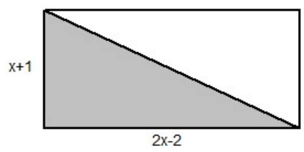
3. $x(x+y) + y(x+y)$

Solution: $$x(x+y) + y(x+y) = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2.$$

4. $a(2a+b) + b(a+2b)$

Solution: $$a(2a+b) + b(a+2b) = 2a^2 + ab + ab + 2b^2 = 2a^2 + 2ab + 2b^2.$$

Exercise 10.3.3 Find an expression for the shaded area of the following figure.



Solution: $$A = \frac{1}{2} b \times h = \frac{1}{2}(2x-2)(x+1) = x^2 - 1.$$

10.4 Practical Exam Questions

Exercise 10.4.1

1. *Solve for x :* $\frac{2}{x} - \frac{4}{5x} = 8$.

Solution:
$$\frac{2}{x} - \frac{4}{5x} = 8 \Rightarrow 20 - 8 = 80x \Rightarrow 80x = 12$$
$$\Rightarrow \therefore\ x = \frac{12}{80} = \frac{3}{20}.$$

2. *Factorise Fully:* $3x - 6y + x^2 - 2xy$.

Solution: $$3x - 6y + x^2 - 2xy = 3(x - 2y) + x(x - 2y) = (3 + x)(x - 2y).$$

3. *Simplify* $\frac{9x^2-4y^2}{6x-4y}$.

Solution:
$$\frac{9x^2 - 4y^2}{6x - 4y} = \frac{(3x - 2y)(3x + 2y)}{2(3x - 2y)}$$
$$= \frac{3x + 2y}{2}.$$

4. *Solve the inequality* $\frac{2x}{3} - 1 \le x + 2$.

Solution:
$$\frac{2x}{3} - 1 \le x + 2 \Rightarrow -1 - 2 \le x - \frac{2x}{3} \Rightarrow -3 \le \frac{x}{3}$$
$$\Rightarrow \therefore\ -9 \le x \text{ or } x \ge -9.$$

5. *Solve for the x:* $\frac{2x+5}{2} - \frac{2}{3} = \frac{2x-1}{4}$.

Solution:
$$\frac{2x + 5}{2} - \frac{2}{3} = \frac{2x - 1}{4} \Rightarrow 6(2x + 5) - 2 \times 4 = 3(2x - 1)$$
$$12x + 30 - 8 = 6x - 3$$
$$12x - 6x = -3 - 22$$
$$6x = -25$$
$$\therefore\ x = -4\frac{1}{6}.$$

Exercise 10.4.2

1. *Simplify:* $\frac{2}{x^2-1} - \frac{3}{x^2-x}$.

Solution:

$$\begin{aligned}\frac{2}{x^2-1} - \frac{3}{x^2-x} &= \frac{2}{(x-1)(x+1)} - \frac{3}{x(x-1)} \\ &= \frac{2x}{x(x-1)(x+1)} - \frac{3(x+1)}{x(x-1)(x+1)} \\ &= \frac{2x-3x-3}{x(x-1)(x+1)} \\ &= \frac{-x-3}{x(x-1)(x+1)}.\end{aligned}$$

2. *Simplify* $\frac{x^2+8x+15}{25-5x} \div \frac{x+3}{x^2-5x}$.

Solution:

$$\begin{aligned}\frac{x^2+8x+15}{25-5x} \div \frac{x+3}{x^2-5x} &= \frac{(x+3)(x+5)}{5(5-x)} \times \frac{x(x-5)}{x+3} \\ &= \frac{\cancel{(x+3)}(x+5)}{5\cancel{(5-x)}} \times \frac{-x\cancel{(5-x)}}{\cancel{x+3}} \\ &= \frac{-x(x+5)}{5}.\end{aligned}$$

3. *Factorise* $x^4 - 256$.

Solution:

$$x^4 - 256 = (x^2-4^2)(x^2+4^2) = (x-4)((x+4)(x^2+16).$$

4. *Simplify* $\frac{(4x-y)^3-4x+y}{4x-y}$.

Solution:

$$\begin{aligned}\frac{(4x-y)^3-4x+y}{4x-y} &= \frac{\cancel{(4x-y)}[(4x-y)^2-1]}{\cancel{4x-y}} \\ &= (4x-y)^2-1 \\ &= (4x-y-1)(4x-y+1).\end{aligned}$$

5. *Simplify* $\frac{x^2+x-2}{x+2} \div \frac{x^2-4x+3}{x^2-3x}$.

Solution:

$$\begin{aligned}\frac{x^2+x-2}{x+2} \div \frac{x^2-4x+3}{x^2-3x} &= \frac{x^2+x-2}{x+2} \times \frac{x^2-3x}{x^2-4x+3} \\ &= \frac{\cancel{(x+2)}\cancel{(x-1)}}{\cancel{x+2}} \times \frac{x\cancel{(x-3)}}{\cancel{(x-1)}\cancel{(x-3)}} \\ &= x.\end{aligned}$$

Exercise 10.4.3

1. *Find the subject of Q for the formula* $4P = 5T + 2Q^2$.

Solution:

$$4P = 5T + 2Q^2 \;\Rightarrow\; 2Q^2 = 4P - 5T$$
$$Q^2 = \frac{4P-5T}{2}$$
$$Q = \sqrt{\frac{4P-5T}{2}}.$$

2. *Make the subject of T for the formula* $B = 2\pi\left(R + \frac{T}{2}\right)$.

Solution:

$$B = 2\pi\left(R + \frac{T}{2}\right) \;\Rightarrow\; R + \frac{T}{2} = \frac{R}{2\pi}$$
$$\frac{T}{2} = \frac{B}{2\pi} - R$$
$$T = 2 \times \left(\frac{B}{2} - R\right)$$
$$\therefore\; T = \frac{B}{\pi} - 2R.$$

3. *If* $w = 2y^3 - 1$, *what is the value of y then* $w = 13$*?*

Solution:

$$\text{Given that } w = 2y^3 - 1 \;\Rightarrow\; 2y^3 = w + 1$$
$$\therefore\; y = \sqrt[3]{\frac{w+1}{2}} = \sqrt[3]{\frac{13+1}{2}} = \sqrt[3]{7}.$$

4. *Rearrange the formula for te area of a annulus,* $A = \pi(R^2 - r^2)$, *to make R the subject.*

Solution:

$$\text{Given that } A = \pi(R^2 - r^2) \;\Rightarrow\; \frac{A}{\pi} = R^2 - r^2$$
$$R^2 = \frac{A}{\pi} + r^2 \;\Rightarrow\; \therefore\; R = \sqrt{\frac{A}{\pi} + r^2}.$$

5. *If* $d = 6t^2$, *find a possible value of t when* $d = 2400$.

Solution:

$$\text{Given that } d = 6t^2 \;\Rightarrow\; t^2 = \frac{d}{6}$$
$$\therefore\; t = \sqrt{\frac{d}{6}} = \sqrt{\frac{2400}{6}} = 20.$$