Year 9 Term 1 Homework Worked Solutions

Student Name: ____________	**Grade**: ____________
Date: ____________	**Score**: ____________

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1 Year 9 Term 1 Week 1 Homework Worked Solutions

1.1 Rational Number

A rational number is a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

1.1.1 Significant figures

A significant figure is a number that is correct within some stated degree of accuracy. The rule for significant figures are:

- All non-zero digits are significant.
- Zeros between non-zero digits are significant.
- Zeros at the end of a decimal are significant.
- Zeros before the first non-zero digit in a decimal are not significant.
- Zeros after the last non-zero digit in a whole number may or may not be significant.

Example 1.1.1 State the number of significant figures in each of these numbers:

1. *2.008*
   Solution: In 2.008, the two non-zero digits are significant and two zeros between these digits are also significant. $\therefore$ the number has four significant figures.

2. *102.50*
   Solution: In 102.50, the three non-zero digits are significant and both the zero in between and at the end of the decimal are significant. $\therefore$ The number has five significant figures.

3. *0.00125*
   Solution: In 0.00125, the three non-zero digits are significant; however, the three zeros at the beginning of the decimal are not significant. $\therefore$ the number has only three significant figures.

4. *9000*
   Solution: In 9000, the non-zero digit is significant. Either some, all or none of the final zeros could possibly be significant. If we knew that the number had been rounded off correct to:

   (a) *1 significant figure, then only the 9 would be significant.*

   (b) *2 significant figures, then only the 9 and the first zero would be significant.*

   (c) *3 significant figures, then only the 9 and the first two zeros would be significant.*

   (d) *4 significant figures, then all the digits would be significant.*

Exercise 1.1.1 Round off 34.535 correct to:

1. 1 significant figure **30**
2. 2 significant figures **35**
3. 3 significant figures **34.5**
4. 4 significant figures **34.54**

Exercise 1.1.2 State the number of significant figures in each of the following:

1. 5002 **4 significant figures**
2. 0.48 **2 significant figures**
3. 3.40 **3 significant figures**
4. 12.0050 **6 significant figures**
5. 0.012003400 **8 significant figures**

Exercise 1.1.3 Round off each of the following correct to 1 significant figure:

1. 325 **300**
2. 280 **300**
3. 2180 **2000**
4. 12.56 **10**
5. 99.45 **100**

Exercise 1.1.4 Round off each of the following correct to 2 significant figures:

1. 8580 **8,600**
2. 123003 **120,000**
3. 8028 **8,000**
4. 0.25349 **0.25**
5. 194.95 **190**

1.1.2 Estimation

- An estimate is an approximate answer that is worked out logically.
- It needs to be of the same order of magnitude.

Exercise 1.1.5 Estimate the answer, as an integer to each of these:

1. $9.6 + 19.3 + 12.2$ $\approx 40 \sim 41$
2. $95.5 - 27.3 + 15.048$ ≈ 85
3. $12.2 \times 3.75 \times 5.4$ $\approx 200 \sim 240$
4. $126.6 \div 9.81$ $\approx 12 \sim 13$
5. $53.5 \div 6.12 \times 8.045$ $\approx 70 \sim 75$

Exercise 1.1.6 Further applications

1. *Evaluate* $\sqrt{4}$ *and* $\sqrt{9}$*, find estimates for the following, correct to 1 decimal place.*
   (a) $\sqrt{5}$ $\approx 2.2 \sim 2.3$
   (b) $\sqrt{8}$ $\approx 2.8 \sim 2.9$
2. *Evaluate* $\sqrt{121}$ *and* $\sqrt{144}$*. Hence , find estimates for the following, correct to 1 decimal place.*
   (a) $\sqrt{125}$ $\approx 11.1 \sim 11.2$
   (b) $\sqrt{145}$ $\approx 12 \sim 12.1$
3. *John decided to re-carpet his lounge room using carpet squares of side length 40 cm. The lounge room is rectangular in shape and measure 4.8 m by 5.6 m.*
   (a) *Estimate the area of the room in square metres.*
   $27\,m^2 \sim 30\,m^2$
   (b) *How many carpet squares are needed to cover an area of 2* m^2.
   $14 \sim 15$
   (c) *Estimate the number of carpet squares that are needed to cover the entire lounge room floor.*
   $225 \sim 250$
   (d) *If the carpet squares are sold in packs of 50 at \$280 per pack, estimate the total cost of the re-carpeting.*
   $5 \times 280 \approx \$1400$

1.1.3 Recurring decimals

- A recurring decimal has an infinite number of decimal places, with one or more of the digits repeating themselves indefinitely.
- Recurring decimals are written with a dot above the first and the last digits in the repeating sequence.
- Every recurring decimal can be expressed as a fraction, so recurring decimals are rational numbers.

Example 1.1.2

1. $0.333333... = 0.\dot{3}$
2. $0.166666... = 0.1\dot{6}$
3. $0.616161... = 0.\dot{6}\dot{1}$
4. $1.329329... = 1.\dot{3}2\dot{9}$

- To convert a fraction to a recurring decimal divide the numerator by the denominator.
- To convert a recurring decimal to a fraction:
  - let the decimal be x
  - multiply both sides by the smallest power of 10 so that the recurring part of the decimal becomes a whole number
  - subtract the first equation from the second and solve the resulting equation.

Example 1.1.3 Convert each of these recurring decimals to a fraction in its simplest form:

1. $0.\dot{6}$
   Solution: let $x = 0.\dot{6}$ *.... (1)*
   $\therefore 10x = 6.\dot{6}$ *.... (2)*
   subtract (1) from (2) we have 9x = 6.
   $\therefore x = \frac{6}{9} = \frac{2}{3}$
2. $0.\dot{1}2\dot{5}$
   Solution: let $x = 0.\dot{1}2\dot{5}$ *........ (1)*
   $\therefore 1000x = 125.\dot{1}2\dot{5}$ *.... (2)*
   subtract (1) from (2) we have 999x = 125
   $\therefore x = \frac{125}{999}$

Exercise 1.1.7 Write each of these as a recurring decimal:

1. 0.6444 ... $= 0.6\dot{4}$
2. 0.31818 ... $= 0.3\dot{1}\dot{8}$
3. 0.3555 ... $= 0.3\dot{5}$
4. 0.919191 ... $= 0.\dot{9}\dot{1}$
5. 0.484848 ... $= 0.\dot{4}\dot{8}$
6. 0.030303 ... $= 0.\dot{0}\dot{3}$
7. 0.029029 ... $= 0.\dot{0}2\dot{9}$
8. 13.95555 ... $= 13.9\dot{5}$

Exercise 1.1.8 Convert each of these recurring decimals to a fraction or a mixed numeral, in simplest form:

1. $0.\dot{3}\dot{5}$

Solution:

$$\text{let } x = 0.\dot{3}\dot{5} \Rightarrow 100x = 35.\dot{3}\dot{5} \Rightarrow 100x - x = 35$$
$$\therefore 99x = 35, \Rightarrow x = \frac{35}{99}.$$

2. $0.4\dot{8}$

Solution:

$$\text{let } x = 0.4\dot{8},\ 10x = 4.\dot{8},\ 100x = 48.\dot{8} \Rightarrow 100x - 10x = 48.\dot{8} - 4.\dot{8} = 44$$
$$\therefore 90x = 44 \Rightarrow x = \frac{44}{90} = \frac{22}{45}.$$

3. $0.\dot{1}4\dot{6}$

Solution:

$$\text{Let } x = 0.\dot{1}4\dot{6} \Rightarrow 1000x = 146.\dot{1}4\dot{6}$$
$$1000x - x = 146 \Rightarrow x = \frac{146}{999}.$$

4. $3.41\dot{6}$

Solution:

$$\text{Let } x = 3.41\dot{6} \Rightarrow 100x = 341.\dot{6} \Rightarrow 1000x = 3416.\dot{6}$$
$$1000x - 100x = 3416.\dot{6} - 341.\dot{6} \Rightarrow x = \frac{3075}{900} = 3\frac{5}{12}.$$

1.1.4 Rates

- A rate is a comparison of two unlike quantities.
- A rate is a measure of how one quantity is changing with respect to another.
- To be in simplest form, a rate must be expressed as a quantity per one unit of another quantity.

Example 1.1.4 Express each of the following statements as a rate in simplest form.

1. *210 km in 3 hours = 70 km/h.*
2. *36 L in 9 min = 4 L/min.*
3. *$180 in 4 hours = $45/h.*

Exercise 1.1.9 Express each of the the following statements as a rate in simplest form:

1. *45 m in 3 seconds* ___***15 m/s***___
2. *260 km in 4 hours* ___***65 km/h***___
3. *$18 for 8 kg* ___***$2.25/kg***___
4. *72 kL in 1.5 hours* ___***48 kL/h***___
5. *26 km on 25 L* ___***1.04 km/L, or 1040 m/L***___
6. *240 heart beats in* $2\frac{1}{2}$ *min* ___***96 beats/min***___

Exercise 1.1.10 Complete the following equivalent rates:

1. *8 cm/s =* ___***480***___ *cm/min*
2. *15 g/min =* ___***900***___ *g/h*
3. *75 cm/s =* ___***45***___ *m/min*
4. *180 kg/h =* ___***4.32***___ *t/day*
5. *142 m/min =* ___***8.52***___ *km/h*
6. *72 km/h =* ___***20***___ *m/s*
7. *2.5¢/mm = $* ___***25***___ */m*
8. *2.8 m/min =* ___***4.032***___ *km/day*

Exercise 1.1.11 Complete the following equivalent rates:

1. 25 m/s = ___ **90** ___ km/h
2. 8 mm/min = ___ **11.52** ___ m/day
3. 0.5 m/min = ___ **0.72** ___ km/day
4. 11 m/mL = ___ **11** ___ km/L
5. 125 m/min = ___ **7.5** ___ km/h
6. 25 mL/s = ___ **90** ___ L/h
7. 720 m/min = ___ **12** ___ m/s
8. 14.6 t/day = ___ **14600** ___ kg/day

Exercise 1.1.12 Convert the following monthly interest rates to annual rates:

1. 0.65% per month ___ **7.8% p.a.** ___
2. 0.8% per month ___ **9.6 % p.a.** ___
3. 1.25% per month ___ **15% p.a.** ___

Exercise 1.1.13 Convert the following annual interest rates to monthly rates:

1. 8% p.a. ___ **0.67% per month** ___
2. 18% p.a. ___ **1.5% per month** ___
3. 4.8% p.a. ___ **0.4% per month** ___

Exercise 1.1.14 Further applications

1. Calculate the daily interest rate on a credit card if the annual rate is 17.8% p.a.
   0.0488%
2. Convert $540/week to an equivalent monthly rate.
   $2340/month
3. Convert $1014/month to an equivalent fortnightly rate.
   $468/fortnight
4. Convert $461.50/quarter to an equivalent weekly rate.
   $35.50

1.1.5 Solving problem with rates

Exercise 1.1.15

1. *George drove 15 km in 10 minutes. At the same speed, how far does he drive in 2 hours?*

   Solution: $15 \times 12 = 180\,km.$

2. *If it takes 3 hours to remove 72 t of sugar from a silo, how long it would take to remove 30 t?*

   Solution: $72 \div 2 = 24\,t/h \;\Rightarrow\; 30 \div 24 = 1\frac{1}{4}\,h$ *or 1 hour 15 min.*

3. *A long distance runner completes a marathon of 42.2 kilometres in 2 hours 15 minutes. Calculate his average speed in km/h and m/s, correct to 2 decimal places.*

   Solution: $2\,h\,15min = 2.25 = 8100\,sec.\ 422000 \div 8100 = 5.21\,m/s = 18.76\,km/h.$

4. *The following currency conversions show the value of 1 Australian dollar (A$1) in US$, euro and NZ$.*

A$1 = US$0.6925	A$1 = 0.5226 euro	A$1 = NZ$1.2171

   Use these currency conversions to convert:

   (a) *A$30 into US$* ***US$ 20.775 = US$ 20.78***

   (b) *A$50 in euro* ***$26.13 euro.***

   (c) *A$500 into NZ$* ***NZ $608.55***

5. *Use the unitary method to answer the following questions:*

   (a) *David paid $4.95 for 3 kg of apples. How much would he paid for 8 kg?*

   Solution: $4.95 \div 3 = \$1.65/kg, \Rightarrow\ 8 \times 1.65 = \$13.20$

   (b) *In a walking race, Peter took 20 minutes to walk 4 km. How long would it take him to walk 15 km?*

   Solution: $4 \div 20 = 200m/min,\ 15000 \div 200 = 75\,min\ or\ 1h15min.$

   (c) *If chicken is being sold for $6.80 per kilogram, find the cost of purchasing 450 grams of chicken.*

   Solution: $6.8 \times 0.45 = \$3.06$

1.2 Miscellaneous Exercises

Exercise 1.2.1 Round off the following correct to 3 significant figures:

1. 99.38 ___**99.4**___
2. 194.63 ___**195**___
3. 499.682 ___**500**___

Exercise 1.2.2 Convert each of these recurring decimals to a fraction or mixed numeral, in simplest form:

1. $0.7\dot{3}$

> **Solution:** Let $x = 0.7\dot{3} \Rightarrow 10x = 7.\dot{3}, \Rightarrow 100x = 73.\dot{3}\ 100x - 10x = 66 \Rightarrow x = \frac{66}{90} = \frac{11}{15}$.

2. $1.\dot{6}\dot{0}$

> **Solution:** Let $x = 1.\dot{6}\dot{0}, \Rightarrow 100x = 160.\dot{6}\dot{0} \Rightarrow 100x - x = 159, \Rightarrow x = \frac{159}{99} = 1\frac{20}{33}$.

Exercise 1.2.3 Complete the following equivalent rates:

1. 25 ¢/cm^2 = \$ ___**2500**___ $/m^2$
2. 160 mL/m^2 = ___**160,000**___ L/km^2
3. 1.5 g/cm^3 = ___**1.5**___ t/m^3
4. \$120/L = ___**12**___ ¢$/cm^3$

Exercise 1.2.4 Further applications

1. At the 1896 Olympic Games, Australia's Edwin Flack won a gold medal in the 800 m in a time of 2 minutes 11 seconds.

   (a) Find the average speed in m/s, correct to 1 decimal place.

   > **Solution:** $800 \div 131 = 6.1\, m/s$

   (b) Express this speed in km/h. ___**21.96 km/h**___

2. On a property sold for \$600,000, a real estate agent receives a commission of \$12,000. At what rate is the commission calculated?

> **Solution:** $\frac{12000}{600,000} \times 100 = 2\%$.

Exercise 1.2.5 Simplify the following:

1. $\frac{(4x-y)^3+4x-y}{4x-y}$

Solution:
$$\begin{aligned}\frac{(4x-y)^3+4x-y}{4x-y} &= \frac{(4x-y)[(4x-y)^2+1]}{4x-y} \\ &= (4x-y)^2+1.\end{aligned}$$

2. $\frac{x^2+x-2}{x+2} \times \frac{x^2-3x}{x^2-4x+3}$

Solution:
$$\frac{x^2+x-2}{x+2} \times \frac{x^2-3x}{x^2-4x+3} = \frac{(x+2)(x-1)}{x+2} \times \frac{x(x-3)}{(x-1)(x-3)} = x$$

3. $\frac{3}{y^2+2y-8} - \frac{2}{y^2+y-6}$

Solution:
$$\begin{aligned}\frac{3}{y^2+2y-8} - \frac{2}{y^2+y-6} &= \frac{3}{(y-2)(y+4)} - \frac{2}{(y+3)(y-2)} \\ &= \frac{3(y+3)}{(y-2)(y+4)(y+3)} - \frac{2(y+4)}{(y+3)(y-2)(y+4)} \\ &= \frac{3y+9-2y-8}{(y+3)(y-2)(y+4)} \\ &= \frac{y+1}{(y+3)(y-2)(y+4)}\end{aligned}$$

4. $\frac{9x^2-4y^2}{6x-4y}$

Solution:
$$\begin{aligned}\frac{9x^2-4y^2}{6x-4y} &= \frac{(3x-2y)(3x+2y)}{2(3x-2y)} \\ &= \frac{3x+2y}{2}.\end{aligned}$$

5. $\frac{2x^2-18}{3x^2+9x}$

Solution:
$$\begin{aligned}\frac{2x^2-18}{3x^2+9x} &= \frac{2(x^2-9)}{3x(x+3)} \\ &= \frac{2(x-3)(x+3)}{3x(x+3)} \\ &= \frac{2(x-3)}{3x}.\end{aligned}$$

Exercise 1.2.6 Factorise the following:

1. $3x^2 - x - 4$

> ***Solution:***
> $$3x^2 - x - 4 = (3x - 4)(x + 1)$$

2. $y^4 - 256$

> ***Solution:***
> $$\begin{aligned} y^4 - 256 &= (y^2)^2 - 16^2 \\ &= (y^2 - 16)(y^2 + 16) \\ &= (y - 4)(y + 4)(y^2 + 16). \end{aligned}$$

3. $x^2 - y^2 - x + y$

> ***Solution:***
> $$\begin{aligned} x^2 - y^2 - x + y &= (x - y)(x + y) - (x - y) \\ &= (x - y)(x + y - 1). \end{aligned}$$

4. $3x - 6y + x^2 - 2xy$

> ***Solution:***
> $$\begin{aligned} 3x - 6y + x^2 - 2xy &= 3(x - 2y) + x(x - 2y) \\ &= (x - 2y)(3 + x). \end{aligned}$$

5. $2x + y^2 + 2y + xy$

> ***Solution:***
> $$\begin{aligned} 2x + y^2 + 2y + xy &= (2x + xy) + (y^2 + 2y) \\ &= x(2 + y) + y(y + 2) \\ &= (y + 2)(x + y). \end{aligned}$$

6. $6y^2 - 13y - 5$

> ***Solution:***
> $$6y^2 - 13y - 5 = (3y + 1)(2y - 5).$$