Year 9 Term 2 Homework Worked Solutions

Student Name: ______________	**Grade**: ______________
Date: ______________	**Score**: ______________

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1 Year 9 Term 2 Week 1 Homework Worked Solutions

1.1 Equations, inequations and formulae

1.1.1 Evaluate the Subject of a Formula

Exercise 1.1.1 Substitution into a formula

1. *If* $A = P(1 + \frac{r}{100})^n$, *find the A when P = 10000, r = 20 and n = 2.*

Solution:
$$A = 10000\left(1 + \frac{20}{100}\right)^2 \Rightarrow \therefore\ A = 14400.$$

2. *If* $T = \frac{n}{2}[2a + (n-1)d]$, *find T if a = 6, d = 3 and n = 8.*

Solution:
$$T = \frac{8}{2} \times [2 \times 6 + (8-1) \times 3] = 132.$$

3. *If* $M = \frac{1}{M_1} - \frac{1}{M_2}$, *find M when* $M_1 = 1.2$ *and* $M_2 = 0.6$

Solution:
$$M = \frac{1}{1.2} - \frac{1}{0.6} = \frac{10}{12} - \frac{10}{6} = \frac{10}{12} - \frac{20}{12} = -\frac{5}{6}.$$

4. *If* $V = \frac{4}{3}\pi r^3$, *find the value of:*

   (a) *r correct to 1 decimal place when* $V = 288\pi\ cm^3$.

Solution:
$$288\pi = \frac{4}{3}\pi r^3 \Rightarrow r^3 = 288 \times \frac{3}{4} \Rightarrow r^3 = 216$$
$$\therefore\ r = 6\,cm.$$

   (b) *r correct to 1 decimal place when* $V = 200\,cm^3$.

Solution:
$$200 = \frac{4}{3}\pi r^3 \Rightarrow r^3 = \frac{3}{4\pi} \times 200$$
$$r^3 = 47.746 \Rightarrow \therefore\ r = 3.6\,cm$$

1.1.2 Changing the Subject of a Formula

Exercise 1.1.2 Make y the subject:

1. $6x - 12 = 3y - 2x$

> **Solution:** $6x - 12 = 3y - 2x \Rightarrow 3y = 6x + 2x - 12 \Rightarrow \therefore\ y = \dfrac{8x - 12}{3}.$

2. $xy - 7 = ax + by$

> **Solution:** $xy - 7 = ax + by \Rightarrow xy - by = ax + 7 \Rightarrow y(x - b) = ax + 7$
> $\Rightarrow \therefore\ y = \dfrac{ax + 7}{x - b}.$

3. $3(4x - 2y) = 15x - 3$

> **Solution:** $3(4x - 2y) = 15x - 3 \Rightarrow 12x - 6y = 15x - 3 \Rightarrow 6y = 12x - 15x + 3$
> $6y = -3x + 3 \Rightarrow \therefore\ y = \dfrac{-3x + 3}{6} = \dfrac{-x + 1}{2}$ *or* $y = \dfrac{x - 1}{-2}.$ *or* $y = \dfrac{1 - x}{2}.$

4. $2y = \frac{3xy}{5} - 4$

> **Solution:** $3(4x - 2y) = 15x - 3 \Rightarrow 10y = 3xy - 20 \Rightarrow 3xy - 10y = 20$
> $\Rightarrow y(3x - 10) = 20 \Rightarrow \therefore\ y = \dfrac{20}{3x - 10}$ *or* $y = \dfrac{-20}{10 - 3x}.$

5. $x^2 = y^2 - 8x$

> **Solution:** $x^2 = y^2 - 8x \Rightarrow y^2 = x^2 + 8x \Rightarrow \therefore\ y = \sqrt{x^2 + 8x}.$

6. $\frac{y}{y-6} = \frac{2x}{3}$

> **Solution:** $\dfrac{y}{y - 6} = \dfrac{2x}{3} \Rightarrow 2x(y - 6) = 3y \Rightarrow 2xy - 12x = 3y$
> $2xy - 3y = 12x \Rightarrow y(2x - 3) = 12x \Rightarrow \therefore\ y = \dfrac{12x}{2x - 3}$ *or* $\dfrac{-12x}{3 - 2x}.$

1.1.3 Inequations

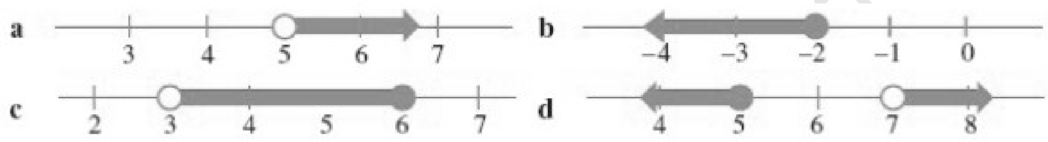
An **inequation** is a number sentence in which two quantities are not equal.
Graphing solutions on the number line:

- place the number that occurs in the solution at the centre of the number line segment.
- draw a closed dot • on this number f the inequality sign is $\geq$ or $\leq$.
- draw an open dot ∘ on this number if the inequality sign is > or <.
- from the dot, draw an arrow along the number line in the direction indicated by the inequality sign.

When multiplying or dividing both sides of an inequation by a negative number, reverse the inequality sign.

Example 1.1.1 Graph each of these inequation on a number line:

a. $x > 5$ *b.* $x \leq -2$ *c.* $3 < x \leq 6$ *d.* $x \leq 5$ *or* $x > 7$



Example 1.1.2

1. $4x + 2 \leq 26$

Solution:

$$4x + 2 \leq 26 \ \Rightarrow \ 4x \leq 24$$
$$\therefore \ x \leq 6.$$

2. $5x + 3 \geq 2x + 12$

Solution:

$$5x + 3 \geq 2x + 12 \ \Rightarrow \ 5x - 2x \geq 12 - 3$$
$$3x \geq 9$$
$$\therefore \ x \geq 3$$

3. $9 \leq \frac{x}{4} + 3$

Solution:

$$9 \leq \frac{x}{4} + 3 \ \Rightarrow \ 6 \leq \frac{x}{4}$$
$$24 \leq x$$
$$\therefore x \geq 24$$

Exercise 1.1.3 Solve the following inequations and graph the solution on a number line.

1. $3x - 1 \geq 14$

   Solution: $3x - 1 \geq 14 \Rightarrow 3x \geq 15 \Rightarrow \therefore x \geq 5.$

2. $19 < 2x + 7$

   Solution: $19 < 2x + 7 \Rightarrow 2x > 12 \Rightarrow \therefore x > 6.$

3. $6 \leq 8(3x - 2)$

   Solution: $6 \leq 8(3x - 2) \Rightarrow 6 \leq 24x - 16 \Rightarrow 24x \geq 22 \Rightarrow \therefore x \geq \dfrac{11}{12}.$

4. $2 - 4x \leq 14 - x$

   Solution: $2 - 4x \leq 14 - x \Rightarrow -3x \leq 12 \therefore x \geq -4.$

5. $\frac{x-2}{4} \leq 3$

   Solution: $\dfrac{x-2}{4} \leq 3 \Rightarrow x - 2 \leq 12 \Rightarrow x \leq 14.$

6. $15 + \frac{x}{2} \geq 8$

   Solution: $15 + \dfrac{x}{2} \geq 8 \Rightarrow 30 + x \geq 16 \therefore x \geq -14$

7. $2 - \frac{x-2}{4} \geq 5$

   Solution: $2 - \dfrac{x-2}{4} \geq 5 \Rightarrow 8 - x + 2 \geq 20 \; -x \geq 10 \Rightarrow \therefore x \leq -10.$

8. $\frac{2x}{3} - \frac{x}{5} \geq 21$

   Solution: $\dfrac{2x}{3} - \dfrac{x}{5} \geq 21 \Rightarrow 10x - 3x \geq 315 \Rightarrow 7x \geq 315 \Rightarrow \therefore x \geq 45.$

1.1.4 Problem Solving

Exercise 1.1.4 Solve the following inequation problems:

1. *If a certain integer is increased by 3 and the result is greater than 7 but less than 13. Find all possible values for the integer.*

Solution: $7 < N + 3 < 13 \;\Rightarrow\; N = 5,\ 6,\ 7,\ 8,\ 9.$

2. *Two sides of a given triangle are 10 cm and 26 cm. What is the range of the possible lengths for the third side of the triangle?*

Solution: *Shortest side* $> 26 - 10 > 16\,cm$; *Longest side* $< 10 + 26 < 36\,cm$.

$\therefore$ *the range of the possible lengths for the third side of the triangle will be* $16 < N < 36\,cm$.

3. *A rectangle is to be constructed with length x cm and width (x-7) cm. The perimeter of the rectangle is to be less than 36 cm. What are the possible values for x?*

Solution:

$$2 \times [x + (x - 7)] < 36 \;\Rightarrow\; 4x - 14 < 36 \;\Rightarrow\; 4x < 50$$

$$\therefore\; x < 12\frac{1}{2}, \text{ but } x > 7,$$

$$\Rightarrow \therefore\; 7 < x < 12\frac{1}{2}$$

4. *The sum of 3 consecutive integers is greater than 9 but no more than 20. What could the integers be?*

Solution:

$$9 < n + (n + 1) + (n + 2) < 20$$

$$\Rightarrow\; 9 < 3n + 3 < 20$$

$$6 < 3n < 17 \;\Rightarrow \therefore\; 2 < n < 5$$

$$\text{that is: } n = \begin{cases} 3,\ 4,\ 5. \\ 4,\ 5,\ 6. \\ 5,\ 6,\ 7. \end{cases}$$

Exercise 1.1.5 Solve the following problems:

1. *Find two numbers such that their sum is 20, while half their difference is 1.*

Solution:

$$\begin{cases} A + B = 20 \\ \frac{A-B}{2} = 1 \end{cases} \quad \begin{cases} A = \frac{20+2}{2} = 11 \\ B = \frac{20-2}{2} = 9. \end{cases}$$

2. *A 2.8 m length of timber is cut into 4 pieces. One piece is twice the length of the shortest piece and the others are 30 cm longer than the shortest piece. Find the length of each piece of timber.*

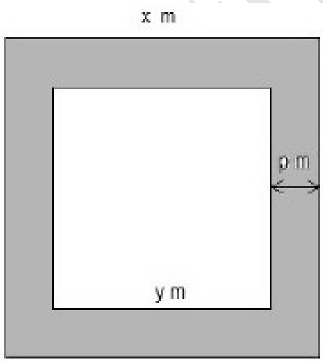
Solution:

$$\text{Let the shortest one be } S \text{ cm:} \;\Rightarrow\; S + 2S + (S + 30) \times 2 = 280,$$

$$5S + 60 = 280 \;\Rightarrow\; S = 44\,cm$$

$$\therefore \text{ the length of each piece of timber are: } 44\,cm,\ 74\,cm\,74\,cm,\ \text{and } 88\,cm.$$

3. *Consider a square garden with sides x metres long. A path p metres wide surrounds a square area of lawn with side y metres, as shown in the figure below:*



(a) *Write down a formula for y in terms of x and p.*

Solution:

$$y = x - 2p$$

(b) *If x = 16 m and p = 2 m, find the area of the path.*

Solution:

$$\text{when } x = 16 \text{ and } p = 2 \;\Rightarrow\; y = 16 - 4 = 12$$

$$\therefore \text{ Area of the path } = x^2 - y^2 = 16^2 - 12^2 = 112\,m^2.$$

1.2 Maths Challenge

Exercise 1.2.1

1. *If* $a\#b = ab - 1$, *find the value of* $\frac{(2\#3)\#5}{2\#(3\#5)}$.

Solution:
$$\begin{aligned}\frac{(2\#3)\#5}{2\#(3\#5)} &= \frac{(2\times 3-1)\times 5-1}{2\times(3\times 5-1)-1}\\ &= \frac{24}{27}\\ &= \frac{8}{9}.\end{aligned}$$

2. *If "&" represents an operation defined as* $x\&y = x^y + y^x$, *find the value of* $(2\&3)\&2$.

Solution:
$$\begin{aligned}(2\&3)\&2 &= (2^3+3^2)^2 + 2^{17}\\ &= 289 + 131072\\ &= 131361.\end{aligned}$$

3. *If* $\frac{x+y}{x-y} = 1\frac{3}{4}$, *find the value of* $\frac{x^2}{y^2}$.

Solution:
$$\frac{x+y}{x-y} = 1\frac{3}{4} \ \Rightarrow\ 4(x+y) = 7(x-y) \ \Rightarrow\ 4x+4y = 7x-7y$$
$$3x = 11y \ \Rightarrow\ \frac{x}{y} = \frac{11}{3} \ \therefore\ \frac{x^2}{y^2} = \frac{121}{9}.$$

4. *Let* $x = \frac{1}{2}(\sqrt[3]{7} - \frac{1}{\sqrt[3]{7}})$, *find the value of* $(x+\sqrt{1+x^2})^3$.

Solution:
$$\begin{aligned}x^2 &= \frac{1}{4}(\sqrt[3]{7^2} - 2 + \frac{1}{\sqrt[3]{7^2}})\\ 1+x^2 &= \frac{1}{4}(\sqrt[3]{7^2} + 2 + \frac{1}{\sqrt[3]{7^2}})\\ \sqrt{1+x^2} &= \frac{1}{2}(\sqrt[3]{7} + \frac{1}{\sqrt[3]{7}})\\ x+\sqrt{1+x^2} &= \frac{1}{2}(\sqrt[3]{7} - \frac{1}{\sqrt[3]{7}}) + \frac{1}{2}(\sqrt[3]{7} + \frac{1}{\sqrt[3]{7}}) = \sqrt[3]{7}\end{aligned}$$
$$\Rightarrow\ x+\sqrt{1+x^2} = \sqrt[3]{7} \ \Rightarrow \therefore\ (x+\sqrt{1+x^2})^3 = 7.$$